Nine Fish

How many different ways can I put 9 fish in 2 bowls? Decide on the factors that you will consider and then solve the problem based upon those factors.

Show all your work and explain why you made the decision you did as you solved the problem.

Nine Fish

Suggested Grade Span

6-8

Task

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Alternate Versions of Task

More Accessible Version:

How many different ways can I put 9 fish in 2 bowls? Use the following chart to record the possibilities.

Possibility #1	# of Fish in Bowl 1	# of Fish in Bowl 2

More Challenging Version:

I have 9 fish. Their names are Aaron, Beatrice, Carlos, Darma, Ellie Mae, Frieda, Gloria, Harry and Jacob. I have 2 fish tanks.

How many different ways can I put my fish into the 2 tanks?

Context

This task can be given to students with some knowledge of number sense and patterns. The student may be able to make some mathematical connections if s/he is familiar with exponents. The task is open-ended, so students should be familiar with setting factors at the beginning of the task that may influence their solution (i.e. type of fish, compatibility of different types of fish, size of tanks, etc.). If students are unfamiliar with fish and fish tanks, it may be necessary to spend a few minutes sharing knowledge about fish or change the context of the problem to familiar animals.

What This Task Accomplishes

The task may be solved on several levels. At all levels, the solution lends itself to the student using and recognizing patterns. In one of the more sophisticated solutions, some students may make a connection between exponents and the pattern. If the connection is made a generalization of the solution is possible (i.e. if there are N fish then there are 2N arrangements).

What the Student Will Do

Students will show different configurations of fish, depending on the assumptions they have made about compatibility, size of tanks and so on. They may make connections to mathematics or other areas of life (like the role of conflict in keeping species apart).

Time Required for Task

45 minutes

Note: This is enough time for students to investigate and write up a solution. However, it can be expanded to allow for interdisciplinary discussions and group time. It would be helpful to spend another 45 minutes to allow students to brainstorm the factors that might affect their solutions (in groups) and share their strategies and solutions.

Interdisciplinary Links

Links can be made to science and social studies. In science, what are some of the factors (like sufficient oxygen) that would affect the number of fish that might be put in one tank? The food chain could also be integrated into the discussion. In social studies, students might discuss the ability of groups or species to live together.

Teaching Tips

A discussion about fish and fish bowls would be appropriate before doing the task to be sure all students understand the context of the task. You may want students who work individually, in pairs or groups, with individual, pair or group responses.

Suggested Materials

- A number of different colored cubes (or other counting materials available)
- Containers that could be used as fish bowls

Possible Solutions

If the student indicates that each fish does not have its own identity and there is no difference between the fish bowls, then there are 10 different ways.

Bowl A: 0, 1, 2, -, -, 9

Bowl B: 9, 8, 7, -, -, 0

If the student indicates that each fish does have has its own identity and the bowls are considered different, then there are 512 or 29 ways.

There are a variety of other solutions depending on the special factors the student sets up in the beginning of the task.

More Accessible Version Solution:

See the solution to the original version.

More Challenging Version Solution:

See the solution to the original version.

Task Specific Assessment Notes

Novice
A task that has no solution or a solution that does not take into consideration nine fish and two tanks and has no evidence of a strategy or explanation of reasoning.

We did not find a piece of student work that fell at the Novice level on this task.

Apprentice
A solution that shows some possible arrangements of nine fish and two fish tanks, but fails to find all possibilities. Some evidence of mathematical reasoning may be inferred, but an incomplete explanation is given.

This first piece of student work shows evidence of a student at the Apprentice level. His/her solution starts with a list of factors that could influence the problem, but the student shows only one possible combination of nine fish in two bowls. His/her solution is not complete; indicating that part of the problem was not understood. Since s/he has nine fish in two bowls, it seems that his/her strategy is partially useful. It leads him/her to one combination, but not to a full solution of the problem. There is some evidence of mathematical reasoning in his/her assumptions in the beginning and the fact that s/he has nine fish distributed in two bowls, but s/he did not go on to consider other possibilities. His/her reasoning is not clear because s/he has no explanation of his/her thinking, although there is some use of appropriate mathematical representation.

Practitioner
A solution that shows the student has a broad understanding of the task. They identify some factors that will influence their solution (compatibility of fish, tank size constraints, etc.), use a strategy with effective mathematical reasoning that leads to a solution, have a clear

explanation, and use appropriate and accurate mathematical representation. This student's solution begins to take into consideration different types of fish and their compatibility. These factors influence his/her solution. The explanation of the solution shows s/he has a broad understanding of the problem. S/he has a strategy that leads to a solution and uses effective mathematical reasoning.

There is a clear explanation of the factors that influenced his/her solution and a clear explanation of his/her strategy and his/her reasoning. His/her mathematical representation is appropriate and accurately labeled.

Expert
This is a solution that shows a deep understanding of the task. The student uses an efficient explanation of strategy and explains their reasons for the decisions they made along the way. Their math representation is actively used as a means of communicating ideas related to the solution of the task. There is precise and appropriate use of mathematical terminology and notation.

This student's solution shows a deep understanding of the problem. S/he gives each fish a separate identity and each bowl is considered different. S/he used a very efficient and sophisticated strategy that leads directly to a solution. S/he starts with simpler cases and recognizes a pattern, which s/he uses to solve the problem.

S/he has a clear and effective explanation detailing how the problem was solved. All the steps were included. S/he explains his/her reasoning so you do not have to infer how and why decisions were made. His/her mathematical representation is accurate and appropriate. His/her representation is actively used as a means of communicating ideas related to the solution of the problem.