Day Lily Dilemma

This fall the teachers had a plant swap. We brought in plants and shared them with each other. I noticed that my day lilies were very crowded. I remembered that 5 years ago I started with 1 day lily. That fall I had 3 plants because the original plant had 2 new "babies." In fact, every year each plant had 2 babies. How many plants did I have in my garden this fall (at the end of the 5th year)?

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Suggested Grade Span

6-8

Task

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Alternate Versions of Task

More Accessible Version:

My day lilies are reproducing at a high rate. Their rate of reproduction is shown below.

Year Number	Number of Day Lilies in Fall
1	3
2	9
3	27

If this continues, how many day lilies will there be in the fall of the 4th year? The 5th year? The 10th year?

More Challenging Version:

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Context

The teachers in my school did have a plant swap - my students saw all the plants that were brought into swap. I explained that although my day lilies probably were not reproducing quite so uniformly, I certainly have many more lilies than I did a few years ago. I told them that sometimes to get a feel for what will happen at a future date, you need to work with what might

be considered an average number of offshoots from each plant. We continue to work with simple equations with one variable. We also were studying exponents.

What This Task Accomplishes

This problem allowed me to assess how well my students are recognizing patterns that could be generalized, as well as recognizing a pattern using exponents. This problem also lends itself to an unusual diagram. Students quickly see that the diagram gets messy to work with and that they have to keep track of the lilies year-by-year in a chart as well. I wanted students to use problem solving to investigate and understand a mathematical concept - exponential growth. I wanted my students to discuss mathematical ideas and make convincing arguments for their solutions. This problem allows kids to appreciate the value of mathematical notation (exponents or multiplying each previous year's lilies by three) and its role in the development of mathematical ideas (communication standard). The task requires students to deal with numbers and number relationships as they investigate the growth of lily plants and exponents. It allows some students that are ready to apply algebraic methods to describe a real-world problem.

What the Student Will Do

Many of my sixth graders needed some "think time" where they really did not do much of anything - a few attempted some strategies. Gradually as they asked some questions (see Teaching Tips) they became more comfortable with the problem. They could begin to either draw a diagram or think about making a chart that kept track of the number of plants at the end of each year. Many diagrams had to be done a few times because beginning attempts were not planned with enough room to grow.

Time Required for Task

My sixth-grade students needed one and a half, 45-minute periods.

Interdisciplinary Links

As we talked about exponential growth, we included a discussion of population and what that kind of graph would look like and why. A science connection would be the growth of body cells, which multiply exponentially.

Teaching Tips

When students are presented with a task and have a question about the problem I often ask the class for suggestions on the answer. For example, I found that many students were confused as to how many plants I had at the end of the first year (three). This should be clear to students so that their solution is not incorrect just because of this misunderstanding. One of my students made it clear to our class by using the analogy that when you are born (year 0) you do not celebrate your first birthday until after the first year is completed.

Another question that was asked was whether the first plant had "babies" the second year. Another student read the part of the problem that said, "In fact every year each plant had two babies," and the class felt comfortable with the answer.

Suggested Materials

Graph paper

Possible Solutions

After the first year there are three plants (original plant and two babies).

After the second year there are nine plants (each of the three plants have two babies + three plants from first year) or you can picture three groups of three plants (the mother and two babies).

After the third year there are 27 plants (each of the nine plants have two babies + nine plants from year two) or you can picture nine groups of three plants.

After the fourth year there are 81 plants.

After the fifth year there are 243 plants.

Generalization: y = year total plants = three

More Accessible Version Solution:

See the solution to the original version.

More Challenging Version Solution:

See the solution to the original version.

Task Specific Assessment Notes

Novice
This student has a limited awareness of the problem. S/he seems to simply be adding two new plants each year. There is little except the diagram to show evidence of a strategy.

Apprentice
This student's diagram shows some understanding that each plant makes two offshoots. However, the student is not successful at finding all the plants that will make offshoots. Somehow s/he thought it was a doubling problem and made that idea fit for the rest of the problem.

Practitioner
This student was successful in making his/her diagram. S/he noticed the tripling effect on the next year. S/he also is trying to make a generalization using some algebra concepts (3 x np = x). Although his/her formula is not a generalization, using algebra to describe a pattern that is observed is a beginning step.

Expert
This student could connect the pattern to exponents. S/he also was comfortable enough with the problem to write a generalization using algebraic notation. The student's chart which breaks up the plants from the babies helps communicate the student's reasoning and strategy. The explanation supports the chart and generalization. The shape of the graph seemed to surprise the student ("Look how much the plant increases each year."). It shows the student is beginning to see and appreciate exponential growth.