

Phone Chain

You have been hired by a local company to design a phone chain that will contact the 100 people in the company in case of emergency.

In order to complete the job:

- Determine how long after you start the chain everyone on the chain would be called if you used a 2, 3, 4 or 5-person chain;
- Explain the advantages and/or disadvantages of using a 2, 3, 4 or 5-person chain;
- Based on the advantages or disadvantages, choose the chain you think would have the greatest possibility of reaching the greatest number of people in the shortest amount of time;
- Present your solution in an organized way that will convince the president of the company that your plan is the best choice.

Exemplars

Phone Chain

Suggested Grade Span

6-8

Task

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Alternate Versions of Task

More Accessible Version:

You have been hired by a local company to design a phone chain that in case of emergency will contact 30 people.

The president of the company will call 2 people, who will then each call 2 people and so on. Each call should take 5 seconds to dial, 28 seconds for ringing, and 25 seconds for talking. Determine the time it will take for all 30 people to be called.

More Challenging Version:

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Exemplars

- Based on the advantages or disadvantages, choose the chain you think would have the greatest possibility of reaching the greatest number of people in the shortest amount of time;
- Present your solution in an organized way that will convince the president of the company that your plan is the best choice.

Write a rule for determining the amount of time it takes to complete a 2-call-per-person phone chain for any number of people that need to be called.

Context

This task allows students to establish the parameters of the problem and requires that s/he make his/her assumptions explicit (e.g. how long each phone call would take). This task can be given to students with some knowledge of number sense. It requires that the student solves the problem for each number of contacts and choose the most efficient number in the chain. It also requires that the student explains clearly and defends his/her solution.

What This Task Accomplishes

This task has two "correct" solutions. (Two and three-person chains each take the same amount of time.) The student will need to make a decision (assuming the "correct" solution) and explain his/her reasoning. The problem lends itself to the student using and recognizing patterns. Some students may make a connection between exponents and the pattern.

What the Student Will Do

Students will show different phone chains (one-person chain, three-person chain, etc.) until they decide which chain is more efficient. The way in which students keep track of their chains will vary. Some students will organize their work in a way that will help them see a pattern.

This task can be accomplished in 45 minutes, although, some students may take more time with more elaborate solutions, while others may take more time to get started. Extra time can be used to establish the context and to discuss phone chains and the solutions students arrive at.

Time Required for Task

1 - 2 hours

Interdisciplinary Links

This is a problem with real-world applications. It might be given in the context of needing to evacuate a very large building or could be tied to evacuating communities and schools during natural disasters, like floods.

Exemplars

Teaching Tips

It is easy to give this problem a real-world context. Perhaps your school has a phone chain for cancellations or delays. With the natural disasters that have been occurring in the last year in the South and Midwest, there are easy connections to be made.

This is a good problem to use to enhance students' understanding of exponents. The solution can be made more concrete by asking students to act out the problem.

Suggested Materials

- Paper
- Pencil
- Calculator

Possible Solutions

Here is a student's solution for a 3-person chain. Others would be solved similarly.

7 rings, 4 seconds in between each ring.

Touch-tone phone:

5 seconds to dial + 28 seconds for ringing + 25 seconds to talk = 58 seconds for each person

# of Round	# of People Who Found Out	# of People Who Know	How Much Time
0	1	1	0
1	3	4	3
2	9	13	3
3	27	40	3
4	81	121	3

It took 12 minutes for a 3-person chain.

More Accessible Version Solution:

# Making Calls	Time it Takes*	# of Calls Made	Total # Called
President	116 seconds	2	2
2	116 seconds	4	6
4	116 seconds	8	14
8	116 seconds	16	30

* (# calls x 58)

Total seconds = 464 seconds ÷ 60 (seconds in a minute) = 7.73 minutes

Phone Chain

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More Challenging Version Solution:

The student first needs to identify how long it will take for 1 complete call. Most will identify a time around 58 seconds.

The rule for determining the # of rounds of calls that needs to be made for a 2-call-per-person phone chain is $[(\text{Total}/2) + 1]/2$.

Take that solution and multiply by 116 seconds, then divide by 60 gives the total number of minutes it takes to contact all people.

Task Specific Assessment Notes

Novice

A task that has no solution or does not take into consideration that the person at the head of the chain begins by calling the number of people in the chain, and that each subsequent caller must be the same number of people.

Apprentice

A solution that evaluates one or two of the chains, but does not evaluate enough chains to arrive at a sound decision. The Apprentice may not keep track of 100 people or use a table to communicate results. The Apprentice may have an incomplete explanation of how they arrived at their decision.

Practitioner

The student has a broad understanding of the problem and the major concepts necessary for a complete solution. S/he has identified constraining factors like the length of a call that affect the results. There is a strategy that leads to a solution and a reasonable explanation. There may be a table or a diagram, but it will not be at the level of detail contained in the Expert solution.

Expert

The student showed a clear understanding of the problem, as s/he identified the special factors that were necessary (e.g. the length of each phone call) to work toward a decision. The student uses an efficient strategy that leads to a solution by setting up a table and keeping track of each element. His/her reasoning at the end when s/he explains when s/he chose the two-person chain shows clear thought. There is a clear and effective explanation detailing how s/he solved the problem. All of the steps are included so the reader does not have to infer how and why decisions were made. The student uses an appropriate and accurate table to organize the data and communicate the results clearly.