Checkerboard Dilemma

How many squares are on a checkerboard?

Be sure to organize your work so that it will be easy for others to follow your thinking and reasoning.

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Suggested Grade Span

6-8

Task

How many squares are on a checkerboard?

Be sure to organize your work so that it will be easy for others to follow your thinking and reasoning.

Alternate Versions of Task

More Accessible Version:

How many squares are on a checkerboard?

Make a table to show the number of each size square. Be sure to organize your work so that it will be easy for others to follow your thinking and reasoning.

More Challenging Version:

How many squares are on a checkerboard? Be sure to organize your work so that it will be easy for others to follow your thinking and reasoning.

Find a way to describe and show the kind of growth that results by comparing the number of each size square. Find the percent each size square is of the total number of squares.

Context

This is a classic problem that I had forgotten was so interesting, especially as I try to get middle school-age students to find generalizations in their mathematics problem solving. It is a problem that is very visual and can be done almost any time during the year since very little arithmetic is needed to be quite successful in solving the problem. However, on the other extreme, it is a great problem to give students who have been introduced to the concept and notation of summation - $\sum$.

What This Task Accomplishes

This task requires students to use problem-solving approaches to investigate, draw logical conclusions and generalize solutions and strategies. It requires understanding and applying reasoning processes, with special attention to spatial reasoning. It encourages students to make connections by exploring problems and describing results using numerical, algebraic and verbal mathematical models or representations.

What the Student Will Do

Most students started with finding the number of small squares. Most knew to multiply the number of rows by the number of columns. However, an interesting discussion revealed a misunderstanding. Some students were concerned that if you multiplied the top row by the first column, that you would be counting the top left square twice. (A classic misunderstanding of actually multiplying eight squares in a row by eight rows.)

Some students did not realize the depth of the problem (that there are more squares than the 64 small squares) - even with prompting that some problems are more involved than what they seem at first. Other students found more squares, but failed to see that some bigger squares would have some of the same small squares in them. It was interesting how students decided to keep track of the squares and how they decided to describe each size square.

Time Required for Task

90 minutes

Two 45-minute periods: One for the kids to initially investigate and begin to organize their work and another period to finalize their arguments.

Interdisciplinary Links

Discussion of traditional game boards and the history of board games.

Teaching Tips

I think it is fair to let kids know that there may be more to this problem than meets the eye. It is a good problem to show them that some problems are more involved than one might think at first, and that sometimes you gain insight into a problem as you are working on it.

I also found it worthwhile to have students share the variety of ways of keeping track of and describing the different sized squares.

Suggested Materials

- Graph paper
- Colored pencils
- Checkerboard

Possible Solutions

8 x 8 = 64 (1 x 1 squares)

7 x 7 = 49 (2 x 2 squares)

6 x 6 = 36 (3 x 3 squares)

5 x 5 = 25 (4 x 4 squares)

4 x 4 =16 (5 x 5 squares)

3 x 3 = 9 (6 x 6 squares)

2 x 2 = 4 (7 x 7 squares)

1 x 1 = 1 (8 x 8 squares)

Generalization:

Let N = number of squares horizontally and vertically

Then: find the sum of (N x N) + (N - 1) x (N - 1) + (N - 2) x (N - 2) .... (1 x 1)

or:

$$\sum_{1}^{n} (N \times N)$$

More Accessible Version Solution:

The solution should be the same as the original task.

More Challenging Version Solution:

The number of squares is: 64 1 x 1; 49 2 x 2; 36 3 x 3; 25 4 x 4; 16 5 x 5; 9 6 x 6; 4 7 x 7 and 1 8 x 8.

So, 64 + 49 + 36 +25 +16 + 9 + 4 + 1 = 204 squares.

The growth can be described as exponential growth and a graph plotting the square numbers will show the exponential growth.

The percent of 1 x 1 squares is 64/204 = .3137254 or about 31%

The percent of 2 x 2 squares is 49/204 = .240196 or about 24%

The percent of 3 x 3 squares is 36/204 = .1764705 or about 18%

The percent of 4 x 4 squares is 25/204 = .122549 or about 12%

The percent of 5 x 5 squares is 16/204 = .0784313 or about 18%

The percent of 6 x 6 squares is 9/204 = .0441176 or about 4%

The percent of 7 x 7 squares is 4/204 = .0196078 or about 2%

The percent of 8 x 8 squares is 1/204 = .0049019 or about .5%

Task Specific Assessment Notes

Novice
This student did not demonstrate an understanding of the complexity of this task and merely counted the squares traditionally noticed. The student uses no math language nor representation to communicate.

Apprentice
The solution is not complete. The student found the 64 small squares and did know that there were other size squares. However, the student's strategy was only partially useful because it did not help them find all the other squares. This student's work did show some mathematical reasoning in finding some of the other size squares. There is some use of representation and some explanation of strategy using some mathematical terminology and notation.

Practitioner
This solution shows the student understood the major concepts needed to find all the squares. The student uses effective reasoning that leads to a solution. There is a clear explanation that uses effective mathematical terminology and notation. The representation is accurate and appropriate for the problem.

Expert
This student shows a deep understanding of the problem by recognizing the pattern and being able to express this pattern algebraically. The explanation, although brief, is concise and to the point. The representation accurately and appropriately communicates the student's solution.