

What's the Problem?

A recent expedition to the North Pole found a message from an explorer of long ago. In the message the explorer talked about an addition problem she was working on, but only the answer to the problem was readable. The explorer said the 2 numbers she added used each digit 1 - 8 only once. The answer that the expedition could read was 7,785.

What was the problem?

Exemplars

What's the Problem?

Suggested Grade Span

3-5

Task

A recent expedition to the North Pole found a message from an explorer of long ago. In the message the explorer talked about an addition problem she was working on, but only the answer to the problem was readable. The explorer said the 2 numbers she added used each digit 1 - 8 only once. The answer that the expedition could read was 7,785.

What was the problem?

Alternate Versions of Task

More Accessible Version:

A recent expedition to the North Pole found a message from an explorer of long ago. In the message the explorer talked about an addition problem she was working on, but only the answer to the problem was readable. The explorer said the 2 numbers she added used each digit 1 - 6 only once. The answer that the explorer could read was 579.

What was the problem?

More Challenging Version:

A recent expedition to the North Pole found a message from an explorer of long ago. In the message the explorer talked about a multiplication problem she was working on, but only the product was readable. The explorer said the 2 factors she multiplied used each digit 1 - 8 only once. The product that the expedition could read was 7,006,652.

What was the problem?

Context

This problem is appropriate for students with knowledge of regrouping in addition. Since there are many possible solutions, students gain experience with problems that have more than one correct answer. Students find this problem so engaging that they will work many addition problems in "solving" it. This problem emphasizes the fact that the commutative property of addition works for individual place values, as well as, complete numbers. This problem-solving approach for standard algorithms works well for subtraction, multiplication and division.

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What This Task Accomplishes

It gives students practice in addition. Students learn that there is often more than one right answer to a problem.

What the Student Will Do

A student may find one solution and stop or continue to find several different solutions, but in either case, students will do many addition problems as they try to arrive at the sum of 7,785 through guess, check and refine.

Time Required for Task

45 minutes

Students who wish to pursue all the solutions may take longer. Others will take longer to get started.

Interdisciplinary Links

The context of this task can be linked to any social studies unit, such as explorers, mapping or archaeology, where one only has fragments of evidence available and the archaeologist or explorer has to reconstruct the context.

Teaching Tips

Allow students to investigate the problem and recognize on their own, that regrouping is needed. Encourage students to look for more than one solution. This is a problem that lends itself to group work.

Suggested Materials

- Paper
- Pencil
- Large graph paper (may be helpful)

Possible Solutions

There are many solutions.

See the Benchmarks.

More Accessible Version Solution:

$123 + 456$ is one possible solution.

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Other combinations are possible that only use one of the six digits. See below for the combinations that make each of the digits.

$$5 = 1 + 4, 2 + 3$$

$$7 = 5 + 2, 6 + 1, 4 + 3$$

$$9 = 6 + 3, 4 + 5$$

More Challenging Version Solution:

One possible solution is 1234 x 5678.

Task Specific Assessment Notes

Novice

The student used inappropriate concepts and procedures in solving the problem. His/her work shows a lack of understanding of place value and the constraints of the problem, only using the digits 1 to 8. S/he uses a strategy that does not lead to a correct solution, using a digit more than one time and using 0 as one of the digits. The mathematical notation indicates a lack of understanding of place value.

Apprentice

The Apprentice found one solution and stopped. S/he uses a strategy that is useful leading to one solution, but does not go beyond that. The mathematical notation is appropriate to the problem. The digits at the top of each problem give some indication of their mathematical reasoning as the check off or circle each digit used.

Practitioner

The Practitioner has a broad understanding of the problem. S/he uses a strategy that leads to multiple solutions. S/he uses effective mathematical reasoning in understanding the commutative property of addition for individual place values ("What I did was I started at the ones column. And started to flip them all the way to the thousands column..."). S/he has a clear explanation of his/her strategy and reasoning. There is effective use of mathematical notation.

Expert

His/her deep understanding of the problem is reflected in the fact that s/he finds multiple answers, indicates that there are still more answers to be found demonstrates a pattern for finding multiple solutions. The strategy used is efficient because s/he knew s/he could find multiple solutions with the eight over the seven in the ones place and could "make a double" of each of those solutions with the seven over the eight in the ones place. His/her explanation is elaborate detailing how the problem was solved. His/her reasoning explained how and why s/he solved the problem the way she did. His/her mathematical terminology and notation are precise, including terms like addend, organized list and double.

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