

Is Dan Losing His Marbles?

Dan played 3 games of marbles. In the 1st game, he lost $\frac{1}{2}$ of his marbles. In the 2nd game he won 4 marbles. In the 3rd game, he won the same number of marbles as he had at the end of the 2nd game. He finished with 32 marbles. How many marbles did Dan start with?

Can you solve this problem in more than one way?

Exemplars

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Suggested Grade Span

3-5

Task

Dan played 3 games of marbles. In the 1st game, he lost $\frac{1}{2}$ of his marbles. In the 2nd game he won 4 marbles. In the 3rd game, he won the same number of marbles as he had at the end of the 2nd game. He finished with 32 marbles. How many marbles did Dan start with?

Can you solve this problem in more than one way?

Alternate Versions of Task

More Accessible Version:

Dan played 3 games with his 32 marbles. In the 1st game, he lost $\frac{1}{2}$ of his marbles. In the 2nd game he won 4 marbles. In the 3rd game, he won twice as many as he did in the 2nd game. How many marbles did Dan end with?

Can you solve this problem in more than one way?

More Challenging Version:

Dan played 3 games of marbles. In the 1st game, he lost the square root of all his marbles. In the 2nd game he won 50% more marbles. In the 3rd game, he won 2 marbles. He finished with 86 marbles. How many marbles did Dan start with?

Can you solve this problem in more than one way?

Context

Although this problem has one correct answer, it can be solved in a variety of ways, depending on the students' background. Some students will easily solve it using an equation if they have had that experience. Others will use diagrams or guess and check. All students should be encouraged to use multiple solutions and to discuss and explain them.

What This Task Accomplishes

This problem requires students to solve for an unknown. Most students at this level will not be able to solve it using an algorithm. The task underlines the importance of reading a problem

Exemplars

carefully and fully understanding it before jumping into the search for a solution.

What the Student Will Do

Students took a number of approaches to solving the problem. Some began with a number and used an equation to arrive at the number of marbles with which Dan finished. If it was not 32 they would try other numbers until they arrived at the correct solution. A few students used figures or manipulatives. Some students went directly to an equation.

Time Required for Task

40 minutes

Interdisciplinary Links

This is a mathematics problem with few apparent interdisciplinary links, although it might be interesting to ask students to make connections to other situations.

Teaching Tips

Discuss the elements of the problem-solving process with your students. Ask your students to be explicit about the assumptions they are making as they begin to solve the problem. If this task is being used as an assessment task, students should have had experience analyzing other complex problems. Students who solve the problem algorithmically should be asked to solve it using another approach. (All students should be asked for multiple solutions.) They should explain how they solved it to assure that they understand mathematical concepts and processes. After students have done the problem, ask them to discuss their varying approaches.

Suggested Materials

- Paper
- Pencil
- Manipulatives (some students might like to use to solve the problem)

Possible Solutions

Dan begins with 24 marbles. In the first game he loses half, so at the end of the game he has 12. In the second game he wins 4 marbles and ends game two with 16 marbles. He wins the same number (16) in the third game and finishes with 32 marbles.

More Accessible Version Solution:

$32 / 2 = 16$ after game 1
 $16 + 4 = 20$ after game 2
 $20 + 8 = 28$ after game 3

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More Challenging Version Solution:

Round	Start	End
1	64	56*
2	56	84**
3	84	86***

* $64 - 8$ (square route of 64)

** 50% of $56 = 28 + 56 = 84$

*** $84 + 2 = 86$

Task Specific Assessment Notes

Novice

The Novice does not know how to begin this problem. S/he has simply drawn the number of marbles with which Dan finished the last game.

Apprentice

This particular Apprentice has a rudimentary understanding of the problem. The strategy is one that could begin to lead toward a solution. There is an incomplete explanation and some use of mathematical notation.

Practitioner

The Practitioner understands the problem and has a strategy that leads to a correct solution. S/he begins by trying one number and when the answer is not correct, moves to another, until the correct answer is found. There is a clear explanation and appropriate use of mathematical notation.

Expert

The use of the equation and the Expert's explanation indicate that s/he has a very clear understanding of the problem and the information necessary for a solution. Using the equation is a very efficient strategy. The explanation is very clear, leading the reader through the steps involved and explaining the strategy and thinking process. While the student uses an equation, s/he obviously uses it with understanding.