

Knitters Club

Frieda Frog, Tracy Turtle, Betsy Bee, Linda Ladybug and Gretchen Gecko were scarf knitters. One night at their knitters club, they had an argument. Each argued that the scarf she knit was the longest. They decided to measure each of the scarves. They decided whoever knit the longest scarf would win an electric knitting machine!

Frieda's scarf was made of 6 connecting scarves. The 1st piece was 4 feet long. The 2nd piece was 1 foot long. The 3rd piece was $\frac{1}{2}$ the 2nd piece. The 4th piece was $\frac{1}{2}$ the 3rd piece, and so on.

Tracy's scarf was made of 6 connecting scarves. The 1st piece was $\frac{5}{8}$ of a foot. The 2nd was $\frac{6}{8}$ of a foot. The 3rd was $\frac{7}{8}$ of a foot, and so on.

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Betsy's scarf was made of 6 connecting scarves. Each of these pieces was $\frac{10}{11}$ of a foot long.

Linda's scarf was made of 10 connecting scarves. The 1st piece was $\frac{1}{11}$ of a foot. The 2nd was $\frac{2}{11}$ of a foot. The 3rd was $\frac{3}{11}$ of a foot, and so on.

Gretchen's scarf was made of 8 connecting scarves. The 1st piece was $\frac{1}{50}$ of a foot long. The 2nd was twice as long as the first. The 3rd was twice as long as the 2nd, and so on.

Rank the creatures in order from the least to greatest total length of scarf knit. Who wins the machine? Show all your work and explain your reasoning.

Knitters Club

Suggested Grade Span

Grades 6–8

Grade(s) in Which Task Was Piloted

Grade 6

Task

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Rank the creatures in order from the least to greatest total length of scarf knit. Who wins the machine? Show all your work and explain your reasoning.

Alternative Versions of Task

More Accessible Version:

Tracy Turtle and Betsy Bee were scarf knitters. One night at their knitters club, they had an argument. Each argued that the scarf she knit was the longest. They decided to measure each of the scarves. They decided whoever knitted the longest scarf would win an electric knitting machine!

Tracy's scarf was made of 6 connecting scarves. The first piece was $\frac{5}{8}$ of a foot. The second was $\frac{6}{8}$ of a foot. The third was $\frac{7}{8}$ of a foot, and so on.

Betsy's scarf was made of 6 connecting scarves. Each of these pieces was $\frac{10}{11}$ of a foot long.

Who won the machine? Show all your work and explain your reasoning.

More Challenging Version:

The original task and...

Determine the area of each scarf if each is 6 inches wide.

NCTM Content Standards and Evidence

Number and Operation Standard for Grades 6–8

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
 - *NCTM Evidence:* Work flexibly with fractions, decimals, and percents to solve problems; and compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.
 - *Exemplars Task Specific Evidence:* This task requires students to determine the sum of fractions and then to compare and order them.

Algebra Standard for Grades 6–8

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Understand patterns, relations, and functions.
 - *NCTM Evidence:* Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
 - *Exemplars Task Specific Evidence:* This task requires students to identify and extend patterns.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher

This is a short to medium length task. The following website has a fraction game your students may enjoy and learn from: <http://illuminations.nctm.org/mathlets/fraction/index.html>

Links

The obvious link is to knitting or crafts.

Common Strategies Used to Solve This Task

Most students will first identify and extend each pattern. Students then compute the totals, and make comparisons either by using number lines or by converting to decimals.

Possible Solutions

Original Version:

Frieda

1 4

2 1

3 1/2

4 1/4

5 1/8

6 1/16

Total $5 \frac{15}{16} = 5.9375$ feet

Tracy

1 5/8

2 6/8

3 7/8

4 8/8

5 9/8

6 10/8

Total $5 \frac{5}{8} = 5.625$ feet

Betsy

$10/11 \times 6 = 5 \frac{5}{11} = 5.45$ feet

Exemplars

Linda

- 1 1/11
 - 2 2/11
 - 3 3/11
 - 4 4/11
 - 5 5/11
 - 6 6/11
 - 7 7/11
 - 8 8/11
 - 9 9/11
 - 10 10/11
- Total $5 \frac{5}{11} = 5$ feet

Gretchen

- 1 1/50
 - 2 2/50
 - 3 4/50
 - 4 8/50
 - 5 16/50
 - 6 32/50
 - 7 64/50
 - 8 128/50
- Total $5 \frac{1}{10} = 5.1$ feet

Frieda's is first longest at 5.9375 feet
Tracy's is second longest at 5.625 feet
Betsy 's is third longest at 5.45 feet
Gretchen's is fourth longest 5.1 feet
Linda's is the shortest at 5 feet.

More Accessible Version:

Tracy

- 1 5/8
 - 2 6/8
 - 3 7/8
 - 4 8/8
 - 5 9/8
 - 6 10/8
- Total $5 \frac{5}{8} = 5.625$

Betsy

$10/11 \times 6 = 5 \frac{5}{11} = 5.45$
So Tracy won.

More Challenging Version:

See the solution to the original version and...

Frieda's is $5.9375 \text{ feet} \times 12 = 71.25 \text{ inches} \times 6 = 427.5 \text{ square inches}$

Tracy's is $5.625 \text{ feet} \times 12 = 67.5 \text{ inches} \times 6 = 405 \text{ square inches}$

Betsy's is $5.45 \text{ feet} \times 12 = 65.4 \text{ inches} \times 6 = 392.4 \text{ square inches}$

Gretchen's is $5.1 \text{ feet} \times 12 = 61.2 \text{ inches} \times 6 = 367.2 \text{ square inches}$

Linda's is $5 \text{ feet} \times 12 = 60 \text{ inches} \times 6 = 360 \text{ square inches}$

Task Specific Assessment Notes**General Notes**

Students will need to be successful with both identifying and extending the patterns, as well as with computing with fractions to achieve a practitioner level with this task.

Novice

The Novice will have no approach to solving the task. Little or no math language will be used. Representations may be attempted but they will not lead to a solution.

Apprentice

A partial solution will be achieved. A fully correct solution may be lacking due to errors in identifying and extending patterns, computing with fractions, or in comparing amounts. Some math language will be used. Some math representations will be attempted.

Practitioner

A correct solution will be achieved with supporting work. Correct reasoning will be evident. Accurate and appropriate math language will be used. Math representations will be used to organize and display the solution.

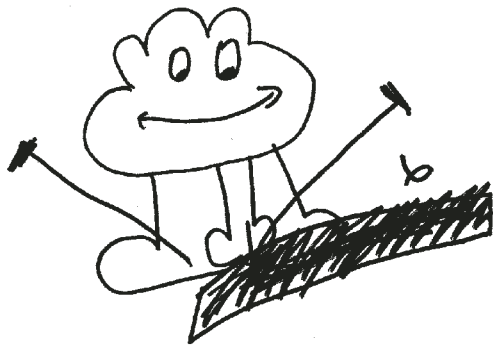
Expert

A correct answer will be achieved. All work will be shown, labeled, and justified. Evidence will be used to support decisions. Connections will be used to extend the solution.

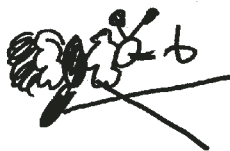
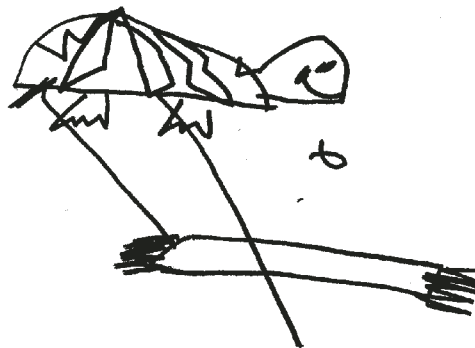
Exemplars

Novice

Many aspects of the task are never addressed.



Little or no problem-solving reasoning is evident.



The student does not advance beyond attempting to identify important information presented in the task.

Exemplars

Apprentice

Some computations are correct, but not all.

I'll see how long there scarves are

friend's scarf

1 4 ft.

2 1 ft.

3 $\frac{1}{2}$ ft. $\frac{8}{16}$

4 $\frac{1}{4}$ $\frac{4}{16}$

5 $\frac{1}{8}$ $\frac{2}{16}$

6 $\frac{1}{16}$

$$\frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} + \frac{15}{16} + 5 \text{ ft} = 5 \frac{15}{16} \text{ ft}$$

Her scarf is $5 \frac{15}{16}$ ft

Tracy's scarf

$$\frac{5}{8} + \frac{6}{8} + \frac{7}{8} + \frac{8}{8} + \frac{9}{8} + \frac{10}{8} = 6 \frac{1}{8}$$

The pattern is the numerator is going up by 1

Betsy's scarf

$$\frac{10}{11} \times 6 = 5 \frac{5}{11}$$

goes up by $\frac{10}{11}$

Linda's scarf

$$\frac{1}{11} + \frac{2}{11} + \frac{3}{11} + \frac{4}{11} + \frac{5}{11} + \frac{6}{11} + \frac{7}{11} + \frac{8}{11} + \frac{9}{11} + \frac{10}{11} = \frac{55}{11} = 5$$

Apprentice cont.

Gretchen's scarf

$\frac{1}{50} \frac{2}{50} \frac{4}{50} \frac{8}{50} \frac{16}{50} \frac{32}{50} \frac{64}{50} \frac{128}{50} = 5 \frac{1}{10}$

1 2 3 4 5 6 7 8

x2 pattern

The student knows to extend the patterns and add the connecting routes, but then does not compare to see who wins.

Exemplars

Practitioner

Math language is relied on to communicate the solution.

Freida

1. 4 ft
2. 1 ft
3. $1/2$ ft
4. $1/4$ ft.
5. $1/8$ ft.
6. $1/16$ ft

total: $5 \frac{15}{16}$
5.9375

Tracy

1. $5/8$ ft.
2. $6/8$ ft.
3. $7/8$ ft.
4. 1 ft
5. $1/8$ ft.
6. $1 \frac{2}{8}$ ft.

$5 \frac{5}{8}$
5.625

Betsy

1. $10/11$ ft
2. $16/11$ ft.
3. $10/11$ ft.
4. $10/11$ ft.
5. $10/11$ ft.
6. $10/11$ ft.

$5 \frac{5}{11}$
5.45

Linda

1. $1/11$ ft.
2. $2/11$ ft.
3. $3/11$ ft.
4. $4/11$ ft.
5. $5/11$ ft.
6. $6/11$ ft.
7. $7/11$ ft.
8. $8/11$ ft.
9. $9/11$ ft.
10. $10/11$ ft.

5

Gretchen

1. $1/50$ ft
2. $2/50$ ft.
3. $4/50$ ft.
4. $8/50$ ft.
5. $16/50$ ft.
6. $32/50$ ft.
7. $64/50$ ft.
8. $128/50$ ft.

$4 \frac{225}{50}$
5

from least to greatest:

person	length
Linda	5'
Gretchen	$5 \frac{1}{50}$ '
Betsy	$5 \frac{45}{110}$ '
Tracy	$5 \frac{625}{1000}$ '
Freida	$5 \frac{9375}{10000}$ '

All work is shown and explained.

Practitioner cont.

Connections

Rule 1! In order to add fractions you need to find common denominators
Rule 2; I turned the fractions into decimals, by dividing the numerator by the denominator.

Relevant observations are made.

A correct solution is achieved.

Exemplars

Expert

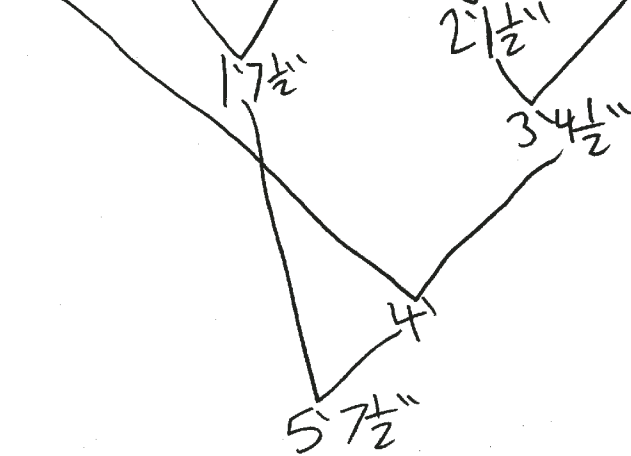
Freida Frog

$$4' + 1' + 6'' + 3'' + 1\frac{1}{2}'' + \frac{3}{4}'' = 5' 11\frac{1}{4}''$$

All work is shown, labeled, and organized.

Tracy turtle

$$7\frac{1}{2}'' + 9'' + 10\frac{1}{2}'' + 1' + 1' 1\frac{1}{2}'' + 1' 3'' = 5' 7\frac{1}{2}''$$



Betsy Bee
 $6 \times 10\frac{1}{2} = 55'$ rounded to 5' 5"

Linda ladybug

$$1 \times 11 = 110 \div 2 = 55 = \frac{55}{11} = 5 \text{ ft.}$$

Precise math language is used throughout.

Exemplars

Expert cont.

The student shows sophistication and a command of fractions and measurement.

Gretchen Gecko

$$\frac{1}{50} + \frac{2}{50} + \frac{4}{50} + \frac{8}{50} + \frac{16}{50} + \frac{32}{50} + \frac{64}{50} + \frac{128}{50} = 5\frac{5}{50}$$

$$\frac{3}{50} \quad \frac{20}{50} \quad \frac{40}{50} \quad \frac{192}{50}$$

$$\frac{60}{50}$$

$$\frac{195}{50} \quad \frac{255}{50} = 5\frac{5}{50}$$

A correct answer is achieved.

Creature	Scarf length	Rank
Freida frog	5' 11 1/4"	1 st place
Tracy turtle	5' 7 1/2"	2 nd place
Betsy Bee	5' 5"	3 rd place
Gretchen Gecko	5 5/50'	4 th place
Linda Ladybug	5'	5 th place

Expert cont.

Connection

To add all of the ^{consecutive} numbers you take the number above the highest number, multiply it by the highest number and divide it by 2.

example:

all of the numbers between 1 and 10
 $10 \times 11 = 110 \div 2 = 55$

Connections extend the solution to Gauss's rule.

