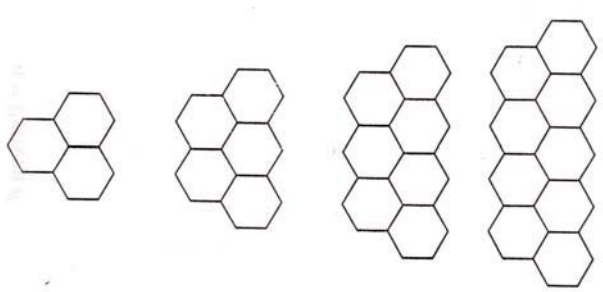
Pattern Block Problem

Mrs. Joslin, the first grade teacher at our school is using hexagonal pattern blocks with her students. They are completing patterns like the ones below:



The student starts with the 1st design and adds onto the first design to get the second design. She expects each child to build to the 10th pattern. How many hexagon pattern blocks will she need for each child? How many hexagon pattern blocks would she need for any number of patterns a child might complete?

Pattern Block Problem

(page 2)

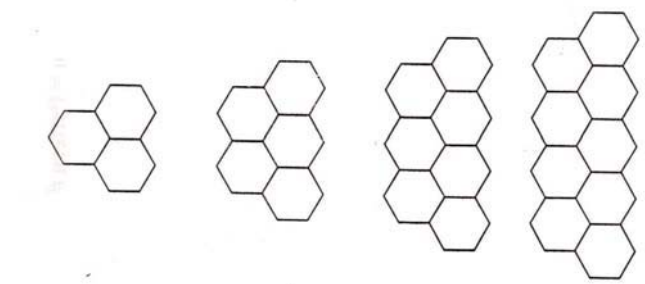
Mrs. Joslin thinks she may need to purchase some additional hexagon pattern blocks for next year. Her class size this year is 10 and next year looks to be 25 students. She currently has 300 hexagonal pattern blocks. If she completes the activity of building 10 patterns next year, how many more pattern blocks will she need to purchase? The hexagons come 2 dozen to a box and cost $12.50. How much money should she put in her budget next year for hexagonal pattern blocks?

Pattern Block Problem

Suggested Grade Span
Grades 6–8

Grade(s) in Which Task Was Piloted
Grades 7 and 8

Task
Mrs. Joslin, the first grade teacher at our school is using hexagonal pattern blocks with her students. They are completing patterns like the ones below:



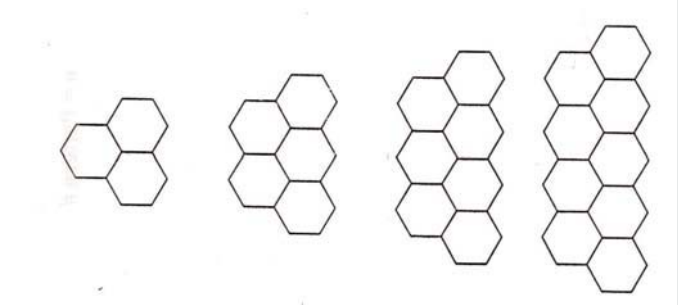
The student starts with the 1st design and adds onto the 1st design to get the 2nd design. She expects each child to build to the 10th pattern. How many hexagon pattern blocks will she need for each child? How many hexagon pattern blocks would she need for any number of patterns a child might complete?

Mrs. Joslin thinks she may need to purchase some additional hexagon pattern blocks for next year. Her class size this year is 10 and next year looks to be 25 students. She currently has 300 hexagonal pattern blocks. If she completes the activity of building 10 patterns next year, how many more pattern blocks will she need to purchase? The hexagons come 2 dozen to a box and cost $12.50. How much money should she put in her budget next year for hexagonal pattern blocks?

Extra Challenge Question:
Mrs. Joslin has a student in her classroom this year who likes to build all of the shapes separately and does not want to add onto the design. How many pattern blocks will the student need if she builds the design 10 times? 100 times?

Alternative Versions of Task

More Accessible Version:
Mrs. Joslin, the first grade teacher at our school is using hexagonal pattern blocks with her students. They are completing patterns like the ones below:



The student starts with the 1st design and adds onto the 1st design to get the 2nd design. She expects each child to build to the 10th pattern. How many hexagonal pattern blocks will she need for each child? How many hexagonal pattern blocks would she need for any number of patterns a child might complete?

More Challenging Version:
You can make the extra challenge question a required part of the task.

NCTM Content Standards and Evidence

Algebra Standard for 6–8

Instructional programs from pre–kindergarten through grade 12 should enable all students to…

- Represent and analyze mathematical situations and structures using algebraic symbols.
  - ***NCTM Evidence:*** Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
  - ***Exemplars Task Specific Evidence:*** This task requires students to analyze and generalize a pattern using algebraic symbols and reasoning.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher
I designed this multi–step task for my algebra students. I used this as an assessment piece after completing activities related to the sum of consecutive numbers and/or Gauss's rule.

Links
This task could link to other tasks that require the application of the sum of consecutive numbers formula. The following web site has additional activities involving consecutive numbers: *http://www.nzmaths.co.nz/PS/L5/Secondary_Units/consecnumbers.htm*

Common Strategies Used to Solve This Task
Most students will approach the problem by making a table to find the rule for the first question in the piece (How many blocks are needed to make the pattern 10 times?). Students will make the table to 10 and then make a rule. They will use a table to verify that the rule works. Most students will realize that they need to purchase a full box of pattern blocks and will round their answers up to the nearest whole box. Students will recognize that this is a consecutive numbers problem and will use Gauss's rule or n(n+1)/2 to solve the final question.

Possible Solutions
Original Version:
How many hexagonal pattern blocks will she need for each child? 21 blocks

How many hexagonal pattern blocks would she need for any number of patterns a child might complete? Rule: 2n+1

If she completes the activity of building ten patterns next year, how many more pattern blocks will he need to purchase? 525–300 = 225 blocks

How much money should she put in her budget for next year for hexagonal pattern blocks? 225/24 (2 dozen) = 9.375 or 10 boxes, 10 boxes x $12.50 = $125.00

How many pattern blocks will the student need if she builds each shape separately to the tenth design? 120 blocks

One hundred times? 10,200 blocks.

More Accessible Version:
See the solution to the original version.

More Challenging Version:
See the solution to the original version.

Task Specific Assessment Notes
General Notes
Most students will organize their data in a table and generate a rule to help solve the problem. There are many parts to this task for students to successfully address before reaching a Practitioner level.

Novice
The Novice may be able to make a table to show how many blocks are needed for the 10th design, but will not be able to proceed from there.

Apprentice
The Apprentice will reach a partial solution. For example, the Apprentice may be able to make a table and write a rule for the first question in the problem, but may not be able to proceed from there. Or, the Apprentice may have an approach to all parts of the task but computation or reasoning errors may lead to an incorrect answer or answers.

Practitioner
The Practitioner will have a correct answer to all parts of the task. All work will be shown and labeled. Algebraic notation and other math language will be used to communicate with the audience.

Expert
The Expert will recognize the task as an appropriate situation to apply Gauss's rule, resulting in an efficient strategy. Precise math language will be used to communicate with the audience. The Expert will accurately address the "extra challenge question".

Novice

Some correct reasoning is present, but many incorrect parts are evident as well.

My teacher handed me a portfolio and said that I had to find out how much Blocks I need for each kid.

The first question I needed to answer is how many block does each child need to create all 10 patterns? If you add up all the total of cubes for each pattern would be 120 pattern block .Then the next question asked how many pattern blocks would she for any number Of patterns a child might complete? 20? 40? 47? 203? It depends on how many pattern They are doing but if you do all 10 patterns then you will need 21 for each child using the same blocks for each pattern.

Pattern#	Total # Hexagons
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

The student makes a table and extends the pattern to the number of hexagons needed for 10, but then cannot proceed successfully from there.

Apprentice

Some awareness of audience is evident.

Some parts are clear.

HEXAGON PROBLEM

Mrs. Joslin's class has a pattern that they want to build on to the same design ten times. I knew that I had to make a table (see below) to find out how many blocks the first kid would need to make 10 patterns. It came out to 21 blocks. I found a rule. I looked at the table and I found out how many blocks the table was going up. It was 2 blocks. I found out if you multiplied the number of the design by 2 and added one it came out to the total number of blocks. My rule is 2x + 1. x equals the number of the design. I could use this to find the amount of blocks for any number of kids.

#Design	blocks
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	**21**

rule

#of design *2+1=#of blocks

The student makes a table and writes a rule for the first question in the problem, but is not able to proceed from there.

The next question asked me how many extra blocks will she need for next year's class of 25 students to make the 10 pattern design. She already has 300 blocks in her class. I multiplied the number of blocks for one kid (21) times 25 kids. The total blocks I needed was 525 then I subtracted 200 and she will need 225 more blocks. I needed to divide 225 by 24 (2 dozen blocks) and that came up to 9.375 boxes of blocks at a cost of $12.50 per box. She will need to buy 10 boxes of blocks for a total cost of $125.00.

Some math language is used to communicate the solution.

Practitioner

Accurate math language is used throughout.

Hexagonal Problem

Date: 4/11/03

When I first received this portfolio piece I realized that it was going to be a real long problem to answer. But our teacher has taught us to take one step at a time with multi-task problems so I did. I figured that I would just look at the first paragraph and answer any problems in there. The first problem it asked was, How many hexagon blocks will she need if each child builds onto pattern one ten times? The other question was how many hexagon blocks would she need for any number of patterns a child might complete. Here is the table to show how I did the problem.

Pattern Number	# Of Hexagons	Rule Check
1	3	(2*1)+1=3
2	5	(2*2)+1=5
3	7	(2*3)+1=6
4	9	(2*4)+1=9
n	2n+1	Rule= 2n+1
10	(2*10)+1=21	

Math representations organize and display the solution.

So as you can see that I used the information that they gave me to make a rule. My rule is 2n+1 with n being the pattern number and 1 being the first pattern you can do. I know my rule works because I used the rule on numbers that I already know the answer to. Then I applied my rule to the first question. The reason I used the number 10 is because the question asked how many blocks for pattern 10.

So the question to number 1 is 21 blocks (Table) and the answer to question two is 2n+1 because she wanted to know for any pattern so she just has to use the rule.

Next I went to the second paragraph. Here are the questions I was asked me three. How many pattern blocks will she need to order with a class of twenty-five when she has 300 blocks? The other was how much is it going to cost Mrs. Joslin if the hexagonal blocks come in packages of 2-dozen at $12.50 a box? Here's how I did the both problems.

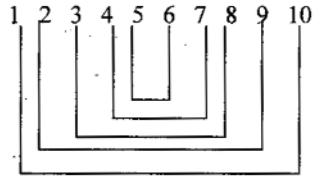
Practitioner cont.

A correct solution is achieved for all parts of the task.

First I went 25 kids* 21 blocks a kid = 525 blocks. Then you subtract 300 from that because she already had 300 blocks so 525-300=225 blocks she needs to order. So the answer to the first question is she will need to order 225 blocks. The way I did the second was 225 blocks / 24 (2- dozen) = 9.375 boxes or about 10 because you cannot order part of a box. So then you multiply 10 * $12.50 = $125.00

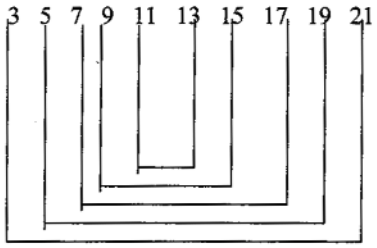
Extra Challenge: She asked how many blocks will Mrs. Joslin need if a child likes to build all the patterns separate for 10 times and 100 times. So here's how I did the fifth question.

Well I used Gauss's Rule. From one to ten there are 5 pairs of numbers. The reason I did from 1 to ten instead of 1 to 11 is because they asked you to ten. Then you use my rule 2n+1 to convert each of the numbers and there are 5 pairs.



The student identifies Gauss's rule and uses it to solve the task.

Then I converted all the numbers and here it is now.



So you can use my rule to convert the numbers one through ten. Then once you do that you add the pairs of numbers together (3 + 21 = 24). Then you multiply that number my how many pairs you have (5). So my answer to how many blocks it will take

Practitioner cont.

to do the pattern, you there is 120 blocks you get that by doing 5 * 24 = 120. Then I had to figure it out for 100.

I know that from 1 to 100 there were 50 pairs. So instead of doing it the long way I just converted pattern 1 to 3 by doing 2 *1 +1 =3. Then I did 100 *2 +1= 201 for the 100 pattern. The reason I did this was because the question asked for 'in 100 patterns how many blocks are there. So to find the number of blocks in a pattern use the rule 2n+1. Then add the largest number to the smallest number (same how many pairs there are. So I added those two numbers together and got 204. Then you have 50 pairs so you, multiply 50 (100/2) by 204 and that equals 10200 blocks.

So here are my answers: 1.) She will need twenty-one blocks per child.

2.) She will need to use my rule 2n+1 (n is pattern #)

3.) She will need to order 225 more blocks.

4.) She should put $125.00 in her budget for hexagons.

5.) The student will need 120 blocks to do it 10 times.

6.) The student will need 10200 blocks to do it 100 times

The approach and reasoning are explained in detail.

Expert

Hexagonal Problem

Today I got a portfolio called Hexagonal Problem. What I had to do was to find the number of hexagons needed to create a certain pattern number. Also, how many more blocks Mrs. Joslin will need and the cost for them all. For extra credit, there is a student in her class that likes to build all the patterns. How many blocks will the student need for ten patterns and a hundred patterns? She has 300 blocks and she is getting 25 new students next year. There are 2 dozen per box and each cost $12.50.

First, I made a table to help sort the information. Look under table #1 for the table and the information on the patterns. Afterwards, I noticed that each time the new pattern comes, you add 2 more blocks from the previous pattern. Using n for a variable for the pattern number, I came up a rule of 2n+1. By using it on the number patterns I already have, I proved that the rule works. Then I used it on ten and got 21 blocks needed for the 10^{th} pattern.

For the second paragraph, I multiplied the 25 students by the 21 blocks needed for 1 child and got 525 bocks Mrs. Joslin needs for her next year class. Next, I subtracted 525 blocks by the 300 blocks she already has and got 225 blocks for her to get. Then, I divided 225 blocks by 2 dozen, or 24 blocks per box, and got 9.375. Because you can't buy a part of a box, I rounded up to 10 boxes Mrs. Joslin will need to buy. Last, I multiplied 10 boxes by $12.50 per box and got a total of $125 for her to put into her budget.

On the extra credit question, I started with Gauss's rule, which was $(n^2+n)/2$. By plugging in the numbers, I came up with a rule of $2[(n^2+n)/2]+n$. Look under information

The student is able to generalize knowledge to achieve a rule for solving for any number of hexagon patterns.

Expert cont.

#1 for how I got the rule. By simplifying it twice, I came up with a rule of n^2+2n. By using it on patterns 2 and 3, I proved that my rule works. For the 10^{th} pattern, I got 120 blocks for a total and for the 100^{th} pattern, I got 10200 blocks for a total.

n= pattern number	Number of blocks	Total number of blocks
1	3	3
2	5	8
3	7	15
4	9	24
n	2n+1	n^2+2n
10	21	120
100	201	10200

2*2=4 4+1=5

2*3=6 6+1 7

2*10=20 20+1=21

2*100=200 200+1=201

Precise math language is used to communicate the solution.

25 students * 21 blocks per student = 525 blocks total

525 blocks – 300 blocks = 225 blocks needed for Mrs. Joslin to buy

225 blocks / 24 blocks per box = 9.375 = 10 boxes needed to buy

10 boxes * $12.50 = $125 needed to save.

Expert cont.

1	2(1)+1=3	
2	2(2)+1=5	3+5=8
3	2(3)+1=7	8+7=15
n	$2[(n^2+n)/2]+n$	

$2[(n^2+n)/2]+n = n^2+n+n = n^2+2n$

$2^2+2*2=8$ blocks total

$3^2+2*3=15$ blocks total

$10^2+2*10=120$ blocks total

$100^2+2*100=10200$ blocks total

The student achieves an accurate and sophisticated solution.