

## Two Pennies

One side of a penny is called the “head” side. The other side of a penny is called the “tail” side. You have 2 pennies. If you toss them on your desk, how many ways can the pennies be “looking” at you?



## Two Pennies

### Suggested Grade Span

Grades Pre K–2

### Grade(s) in Which Task was Piloted

Grades 1 and 2

### Task

One side of a penny is called the “head” side. The other side of a penny is called the “tail” side. You have 2 pennies. If you toss them on your desk, how many ways can the pennies be “looking” at you?

### Alternative Versions of Task

#### *More Accessible Version:*

This is a basic task and does not require a more accessible version.

#### *More Challenging Version:*

One side of a penny is called the “head” side. The other side of a penny is called the “tail” side. You have 2 pennies. If you toss them on your desk, how many ways can the pennies be “looking” at you?

Now take 1 penny and toss it on your desk 20 times. Record which side is looking at you each time. How many “heads” and “tails” did you get? What can you say about what happens when you toss a penny?

### NCTM Content Standards and Evidence

#### Data Analysis and Probability Standard for Grades Pre K–2

Instructional programs from pre–kindergarten through grade 12 should enable students to —

- Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them
  - *NCTM Evidence:* Sort and classify objects according to their attributes and organize data about the objects
  - *Exemplars Task Specific Evidence:* This task requires students to find all the different arrangements of two pennies.

#### Time/Context/Qualifiers/Tip(s) From Piloting Teacher

This task is considered a medium length task and it took my students one class period in which to complete it. You may want to have paper plates for the pennies to be flipped onto. The paper plate muffles the noise of coins hitting the desk and avoids pennies falling off the desk. You might also demonstrate shaking the coins in your hand and then dropping them on the paper plate so coins are not flipping in the air.

## Links

This task may complement activities relating to the study of coins.

## Common Strategies Used to Solve This Task

Most students started by tossing the coins on their desk to see what sides of the pennies were facing them. They then recorded what they saw and went on to find a different combination and continued recording new events.

## Possible Solutions

### *Original Version:*

There are four different events: heads, heads; heads, tails; tails, tails; and tails, heads. Remember that tails,heads and heads,tails are different events.

### *More Accessible Version:*

Refer to the original version solution.

### *More Challenging Version:*

The first part is the same as the original version. Hopefully, the results of the penny toss will be close to 10 heads and 10 tails. The conclusion should be that it is just as likely to get a head as a tail or that the number of heads and tails are even or nearly even.

## Task Specific Assessment Notes

### General Notes

This task encourages the use of diagrams especially the tree diagram if students have been introduced to it.

### Novice

The Novice will not have a strategy that will allow them to find all the combinations. Any arguments made will have no mathematical basis. They may not be able to construct a mathematical representation of the “heads” or “tails”.

### Apprentice

The Apprentice will be able to choose a partially correct strategy that may lead to some but not all of the combinations. Some correct reasoning is present. They will make an attempt at constructing a mathematical representation.

### Practitioner

The Practitioner will choose a strategy that will lead to finding all the combinations. They will construct an appropriate and accurate mathematical representation. A sense of audience or purpose is communicated.

## Expert

The Expert will have all that a Practitioner has and more. They may go on to experiment with one or two of the pennies. They may toss one coin and record the results and find that it is as likely to toss a “head” as it is a “tail”. They may toss both coins and record the results. They may notice and question why there are more “heads,tails” (or “tails, heads”) than “tails, tails” or “heads, heads”. They may go on to take three pennies and find the number of combinations.

The student does not  
mathematically engage  
in the task.

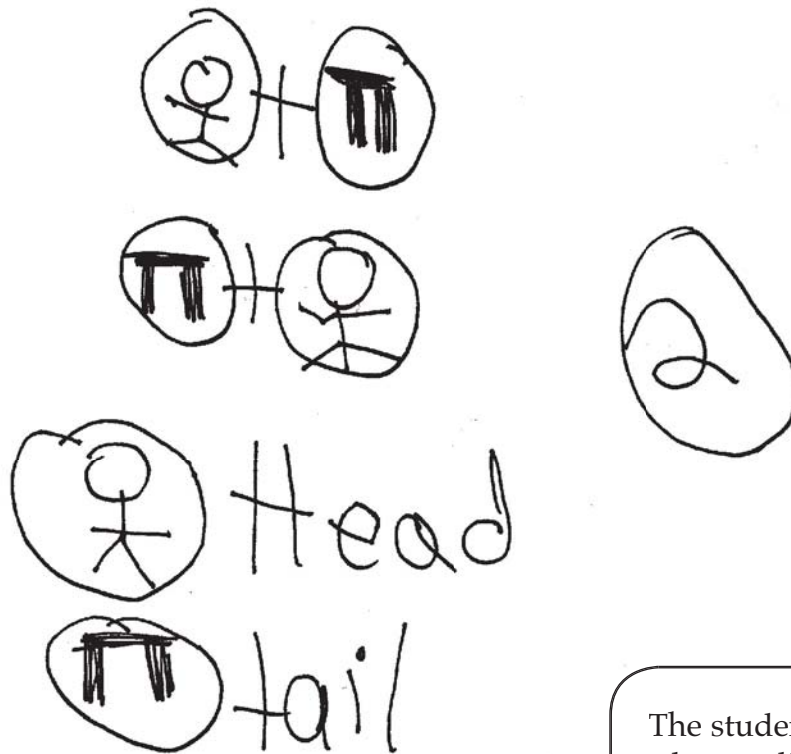


2 pennies

---

# 9 Yādūg

## Apprentice

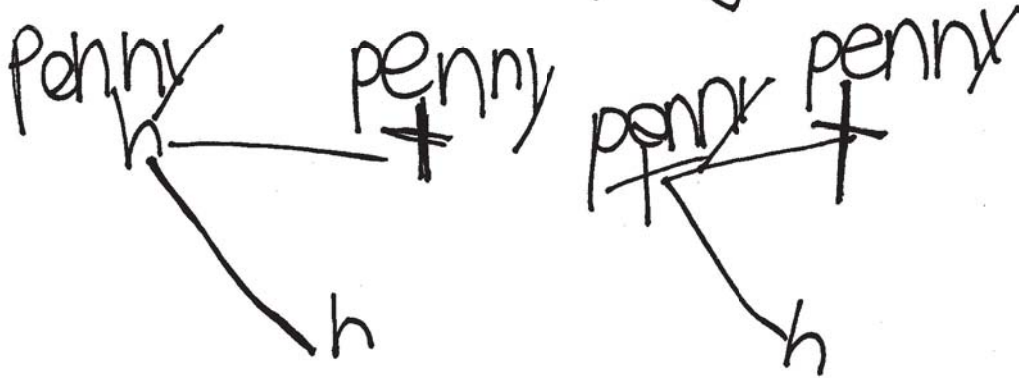


The student demonstrates understanding of the concept of two pennies, "looking at you", but fails to find all the combinations.

An observation that, "half the coins show heads and half are tails" is made.

Practitioner

tree diagram



The student used a tree diagram, which communicates the approach.

A correct solution is achieved.

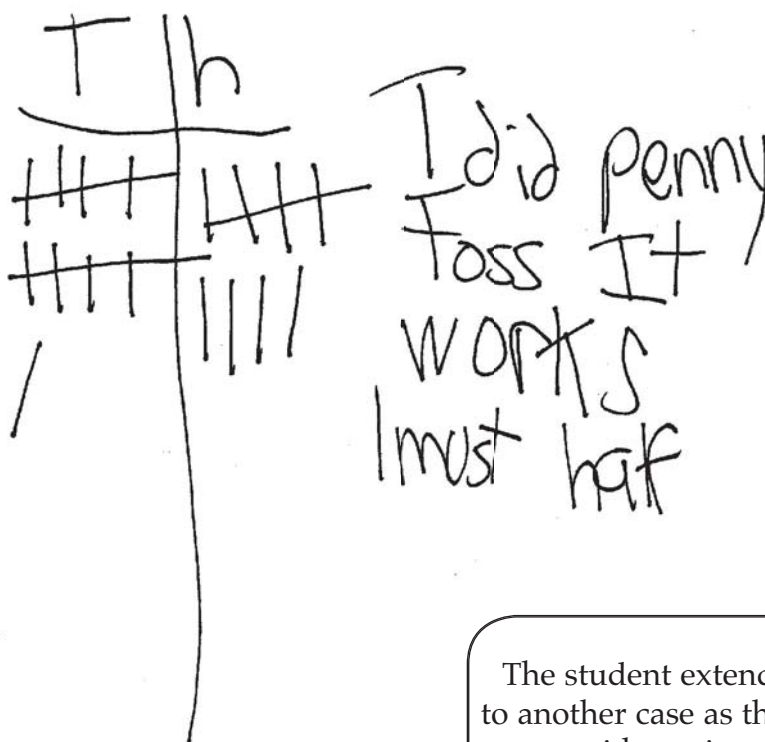
4 combinations  
haf is h haf st



Expert cont.

The student made an observation about the toss of a penny.

I did 3 ways  
 I got 4 combinations  
 all the time



The student extended the solution to another case as they experimented with tossing one penny.