

Painting Plans

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Amy got busy. First, she measured her room. She found that each wall of her square room was 12 feet long on each side, by 8 feet high. She had 2 windows that were 3 feet by 5 feet and obviously would not be painted. Her closet door was 3 feet by 6 feet and that would not be painted either. She did not bother to measure her bedroom door because that would be painted red.

Amy went to the paint store to find out the cost of the materials she needed. She was told that she would have to use 2 coats of paint for the walls to look as bright as she wanted. Here is what Amy learned:

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A quart of red paint covers 75 square feet and costs \$5.50.

A roller-brush and pan cost \$6.00.

A small brush costs \$2.50.

A plastic drop-cloth costs \$3.75.

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Painting Plans

Suggested Grade Span

Grades 3–5

Grade(s) in Which Task Was Piloted

Grade 5

Task

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Alternative Versions of Task

More Accessible Version:

Amy loves the color red and decided to paint her bedroom walls bright red. First, she measured her room. She found that each wall of her square room was 12 feet long by 8 feet high. She also had 2 windows that were 3 feet by 5 feet and would not be painted. She did not bother to measure her bedroom door because that too would be painted red. A gallon of red paint covers 300 square feet and costs \$15.00. How many cans of paint will she need to buy to paint her room?

Show all your mathematical thinking.

More Challenging Version:

The original version and...

After painting her walls red, Amy decided to paint the trim black. The trim went around the perimeter of the bottom of her room, and around the windows, bedroom door, and closet door. Since she now needed the measurements of the bedroom door, she measured it and found that it was $7\frac{1}{2}$ feet high and 4 feet wide. The trim is $3\frac{1}{2}$ inches wide. The paint for the trim comes in the same size containers at the same prices as the paint she used for the walls. Approximate the cost of painting the trim.

NCTM Content Standards and Evidence

Geometry Standard for Grades 3–5

Instructional programs from pre-kindergarten through grade 12 should enable all students to...

- Use visualization, spatial reasoning, and geometric modeling to solve problems.
 - *NCTM Evidence:* Use geometric models to solve problems in other areas of mathematics, such as number and measurement.
 - *Exemplars Task Specific Evidence:* This task requires students to visualize the bedroom with 4 walls, 2 windows, and 1 closet door so square footage can be determined.

Measurement Standard for Grades 3–5

Instructional programs from pre–kindergarten through grade 12 should enable all students to...

- Apply appropriate techniques, tools, and formulas to determine measurements.
 - *NCTM Evidence:* Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms.
 - *Exemplars Task Specific Evidence:* This task requires students to understand that they need to find the area of the bedroom walls minus the area of the windows and closet door to determine the amount of paint needed for the project.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher

This is a medium to long length task.

Links

The following web site has a calculator program for helping plan painting projects that students may enjoy experimenting with or use for verifying solutions:

<http://www.flexbon.com/calc.shtml>

This website will allow a student to experiment with how area changes as perimeters change:

<http://www.shodor.org/interactivate/activities/perm/index.html>

Common Strategies Used to Solve This Task

Most students will use diagrams supported by charts and computation to achieve a solution.

Possible Solutions

Original Version:

Area of each wall: $12 \times 8 = 96$ square feet $\times 4$ walls = 384 square feet

Area of windows: $3 \times 5 = 15 \times 2 = 30$ square feet

Area of closet door: $3 \times 6 = 18$ square feet

$384 - 48 = 336$ square feet of paint needed $\times 2$ coats = 672 square feet

Cost per cup of paint

1 gallon = 16 cups, $\$15.00 \div 16 = \0.9375 cents per cup

Half-gallon = 8 cups, $\$8.00 \div 8 = \1.00 per cup

1 quart = 4 cups, $\$5.50 \div 4 = \1.375 per cup

Therefore, it is least expensive to buy paint by the gallon.

Exemplars

Painting Accessories: roller \$6.00 + small brush \$2.50 + drop cloth \$3.75 = \$12.25

$\$50.00 - \$12.25 = \$37.75$ left for paint

2 gallons (600 square feet) = \$30.00

1 quart (75 square feet) = \$5.50

The project would cost \$47.75.

Amy would have \$2.25 left!

More Accessible Version:

Area of each wall: $12 \times 8 = 96$ square feet $\times 4$ walls = 384 square feet

Area of windows: $3 \times 5 = 15 \times 2 = 30$ square feet

$384 - 30 = 354$ square feet of paint needed, so two gallons of paint would be needed.

More Challenging Version:

12 feet $\times 4$ walls = 48 feet (perimeter of room)

7.5 feet high bedroom door $\times 2 = 15$ feet for door (top of door is covered in perimeter)

6 feet high closet door $\times 2 = 12$ feet for door (top of door is covered in perimeter)

$3' + 3' + 5' + 5' = 16$ feet $\times 2$ windows = 32 feet

$48' + 15' + 12' + 32' = 107' \times 12'' = 1284'' \times 3.5''$ width = 4494 square inches which is equal to approximately 31.21 square feet, so only a quart of black paint is needed.

Task Specific Assessment Notes

General Notes

This task requires students to manipulate many pieces of information, to decide what information is needed and when, and then to execute computations correctly.

Novice

The Novice will have a rudimentary understanding of the task, but it will not lead to even a partially correct solution. Little or no correct reasoning or justification of work shown will be evident. Little or no math language will be used, or it will be used incorrectly.

Apprentice

The Apprentice will achieve a partially correct solution, but omissions, computation errors, or reasoning errors will lead to an incorrect solution. For example, the student may neglect to address two coats of paint, and/or forget to take into consideration the windows and/or closet door. Some math language may be used correctly and some correct reasoning may be present. An attempt at math representations will be used to communicate the solution and assist with understanding.

Practitioner

The Practitioner will achieve a correct solution and all work will be shown and labeled. All parts of the task will be successfully addressed and representations will help organize and display the solution. Math language will be used to communicate the solution and mathematically relevant observations will be made.

Expert

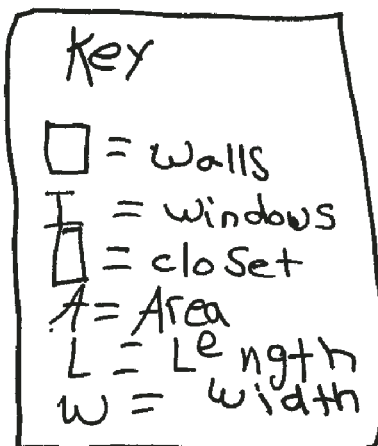
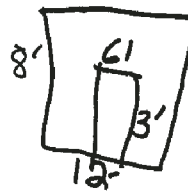
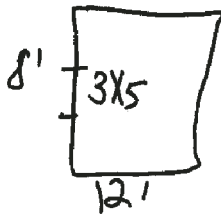
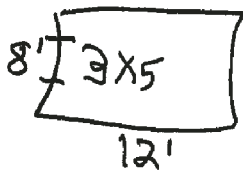
The Expert will clearly label and organize all of their work. Math representations and language will clarify thinking and communicate with the audience the approach and reasoning used. A correct solution will be achieved and math connections will extend the solution.

Exemplars

Novice

The student then subtracts 243 square feet, and there is no evidence of how or why this amount has been derived.

I have to find the area and make the table of the problem of what we have to do to figure it out. I will make a diagram of Amy's room.



$$\begin{array}{r} 384 \text{ 59 feet} \\ - 243 \text{ 59 feet} \\ \hline 141 \text{ 59 feet} \end{array}$$

A correct total of square feet is shown, but it is unclear how it was determined.

It appears the student has some understanding of the task, but after looking more closely, it is clear that although numbers from the task are used to label diagrams, the student has no strategy of how to use those numbers.

Exemplars

Apprentice

The cost of the paint is not considered.

I need to find out if Amy will be able to paint her room bright red for ^{upper the} cost to \$50.00. To figure out my answer is a bit gay and a table.

She has 336 sq' totle to paint.

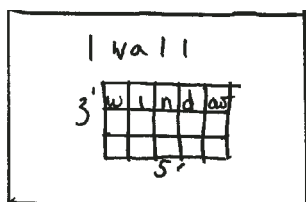
I am going to add up \$6.00 + \$2.50 + \$3.75 to see if she will have enof monny to bye all of the stupe. The answer I got is \$12.25

gallon	sq'
1	300 sq'
2	600 sq'
3	1200 sq'

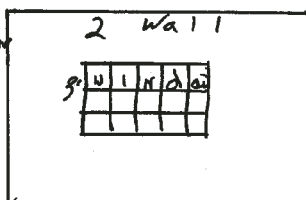
The student has some correct work, but neglects to consider that two coats of paint are needed.

Exemplars

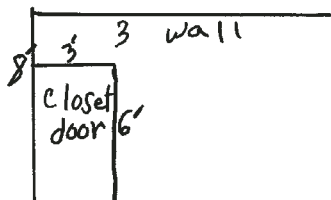
Apprentice cont.



12'
same as
wall 2



12'
 $3 \cdot 5 = 15$
 $A = 15 \text{ sq}'$
 $\begin{array}{r} 96 \\ - 15 \\ \hline 81 \text{ sq}' \end{array}$



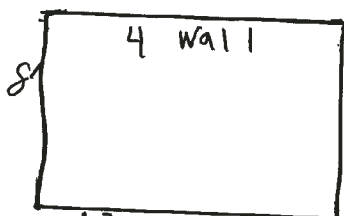
12' $\frac{8}{16}$
 $3 \cdot 6 = A$
 $A = 18 \text{ sq}'$
 $\begin{array}{r} 96 \\ - 18 \\ \hline 78 \text{ sq}' \end{array}$

TABLE

WALL	area
4	96 sq'
3	78 sq'
2	81 sq'
1	81 sq'

total
336 sq'

Key
A = Area
L = Length
W = width
A = L \cdot W



$\begin{array}{r} \times 12 \\ 8 \quad 12 \\ \hline A = 96 \text{ sq}' \end{array}$

A chart is used to organize aspects of the solution. Some correct math language is used.

More paint than needed for the student's incorrect solution is "ordered".

Practitioner

All work is shown, organized, and labeled.

Math language is used throughout to communicate the solution.

8'

12'

3'

6'

Door

$$\begin{array}{r} 12 L \\ \times 8 W \\ \hline 96 \text{ sq}' \end{array} \quad \begin{array}{r} 6 W \\ \times 3 L \\ \hline 18 \text{ sq}' \end{array}$$

$$\begin{array}{r} 96 \\ - 18 \\ \hline 78 \text{ sq}' \end{array}$$

A = Area
W = Width
L = Length

A = L · W
A = 8' · 12'
A = 96 sq'

A = 78 sq'

Wall 2

8'

8'

5'

3'

Window

$$\begin{array}{r} 12 L \\ \times 8 W \\ \hline 96 \text{ sq}' \end{array} \quad \begin{array}{r} 5 W \\ \times 3 L \\ \hline 15 \text{ sq}' \end{array}$$

$$\begin{array}{r} 96 \\ - 15 \\ \hline 81 \text{ sq}' \end{array}$$

A = 81 sq'

Wall 3

8'

12'

5'

3'

Window

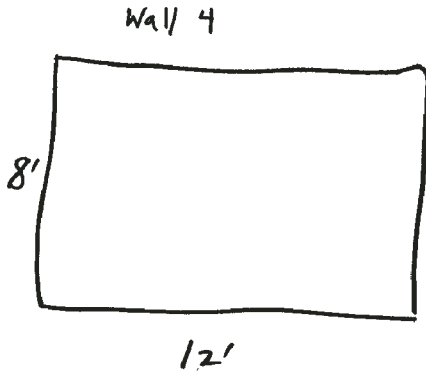
$$\begin{array}{r} 12 L \\ \times 8 W \\ \hline 96 \text{ sq}' \end{array} \quad \begin{array}{r} 5 W \\ \times 3 L \\ \hline 15 \text{ sq}' \end{array}$$

$$\begin{array}{r} 96 \\ - 15 \\ \hline 81 \text{ sq}' \end{array}$$

A = 81 sq'

Exemplars

Practitioner cont.



$$\begin{array}{r} 12 \text{ L} \\ \times 8 \text{ W} \\ \hline 96 \text{ sq}' \end{array}$$

$A = 96 \text{ sq}'$

Math representations assist in problem solving and portraying the solution.

$$\begin{array}{r} \text{wall 1} = 78 \\ \text{wall 2} = 81 \\ \text{wall 3} = 81 \\ \text{wall 4} = 96 \\ \hline 336 \text{ sq}' \end{array}$$

$$\begin{array}{r} 336 \\ \times 2 \\ \hline 672 \text{ sq}' \text{ for 2 coats} \end{array}$$

$A = 336 \text{ sq.}'$ needed

Table

Materials	cost
2 gallon	\$30.00
1 quart	\$5.50
roller brush and pan	\$6.00
small brush	\$2.50
plastic drop cloth	\$3.75

$$\begin{array}{r} \$30.00 \\ \$5.50 \\ \$6.00 \\ \$2.50 \\ \$3.75 \\ \hline \$47.75 \end{array}$$

$$\begin{array}{r} \$19.91 \\ \$50.00 \\ \$47.75 \\ \hline \$21.16 \text{ left over} \end{array}$$

$3 \text{ sq}'$ paint left over

A correct solution is achieved.

Expert

All work is shown to support a correct solution.

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A/R	Con.	Sol.	Lan.	Rep.	Doc.

Exemplars

Expert cont.

I need to find out if Amy can
have a red room for under \$50.00

I am going to use a diagram and
a table.

Exemplars

Expert cont.

Formulas for finding area are shown.

The solution is organized and well labeled.

Wall 1

5'
12'

$5 \times 12 = 15$
96
-15
81 sq'

81 96
↑
Wall

Wall	Area
1	81 sq'
2	81 sq'
3	78 sq'
4	96 sq'

Wall 2 is the same

Wall 3

2'
6'
12'

96 Wall
18 Closet Door
81 78

Wall 4

96 sq' Wall

81 336
81 x 2
78 672 sq
+ 96 2 coats
336 sq'
on first coat

Exemplars

Expert cont.

Mathematically relevant observations about extra paint, and the fractional relationship between a quart and a gallon help further the solution.

~~$$\begin{array}{r} 300 \\ + 300 \\ \hline 600 \end{array}$$~~

$$\begin{array}{r} 300 \\ + 300 \\ \hline 600 \\ + 75 \\ \hline 675 \end{array}$$

$$\begin{array}{r} \$30.00 \leftarrow 2 \text{ gallons} \\ + 5.50 \leftarrow 1/4 \text{ gallons} \\ \hline \end{array}$$

$$35.50 \leftarrow \text{roller brush}$$

$$\begin{array}{r} + 6.00 \checkmark \\ \hline 41.50 \end{array}$$

$$\begin{array}{r} + 2.50 \leftarrow \text{Small Brush} \\ \hline 44.00 \end{array}$$

$$\begin{array}{r} + 3.75 \leftarrow \text{drop cloth} \\ \hline 47.75 \end{array}$$

Connections

$$50.00$$

$$\begin{array}{r} 47.75 \\ \hline \end{array}$$

$$2.25 \text{ left over}$$

2 gallons and 1/4

675 sq' that can be covered

672.5 sq' that need to be covered

3 sq' be covered
 ↑
 enuf paint left to paint 3 sq'

Formula

$$L \cdot W = A$$

Key

L = length
 W = width
 A = Area

Precise math language and representations are used to communicate with the audience the approach and reasoning used.