

Big Scoop

(Adapted from Connected Mathematics)

Molly and Devon bought ice cream at Ben and Jerry's. They thought about bringing one home to their brother Bailey, but decided the ice cream would melt before they could get it to him. Not to worry! Molly thought that if they got the ice cream in a cup, even if it melted, it would fill the cup exactly. Devon thought that if they got ice cream in a cone, even if it melted, it would fill the cone exactly. Each container had a height of 8 cm and a radius of 4 cm. Each scoop of ice cream was a sphere with a radius of 4 cm. Which person, if either, is right?

Grade Level 6 – 8

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Context

This task was given to students during a geometry unit on volume. Prior to being assigned this task, the students had “discovered” formulas for the volumes of cylinders, cones, and spheres by constructing each out of graph paper (cylinder), plastic film (cone), and play dough (sphere).

What This Task Accomplishes

This task can be used as an assessment piece for a unit on volumes.

Time Required for Task

Most students completed this task within a 45 minute class period.

Interdisciplinary Links

Science

Students bring to this task knowledge they have gained in science for finding volume. It also provides an opportunity to investigate density.

Social Studies

Students could surf the Internet to learn more about ice cream manufacturing, marketing, and distribution.

Teaching Tips

This task can be made more or less complicated by changing the numbers presented in the task or the shapes of the containers.

Suggested Materials

calculators, paper, actual cones and cups.

Possible Solutions

The approximate volumes (depending on the value of π used):

cone:	134 cm ³
cup:	402 cm ³
ice cream:	268 cm ³

If the ice cream melted, it will overflow the cone, but will fill 2/3 of the cup.

Benchmark Rubric

Novice	<ul style="list-style-type: none"> • No work is present, as the student seems to merely have made a guess. • No math language is used. • No mathematics reasoning is evident.
Apprentice	<ul style="list-style-type: none"> • The solution has an approach that works for part of the problem. • An attempt is made to calculate the volumes of ice cream, cone, and cup, but the attempt does not lead to a correct answer. • Some math language is used. • No math representation is attempted.
Practitioner	<ul style="list-style-type: none"> • The volumes of ice cream, cone, and cup are correctly calculated. • A correct solution is achieved. • The approach and reasoning are evident. • Appropriate and accurate math language is used.
Expert	<ul style="list-style-type: none"> • The volumes of ice cream, cone, and cup are correctly calculated. • A correct solution is achieved. • The approach and reasoning is explained. • Appropriate and accurate math language is used throughout. • Representations aid in understanding the solution. • Mathematically relevant comments or connections are made.

Author

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Novice

Cone or cup?

In this problem we had to figure out if Devon let the ice cream she bought for her sister melt and see if the cup or the cone will fill up to the top and which one would over flow. After calculating and figuring I found that the cone over flowed but the cup was to big and didn't quite fill up.

No work is shown.

No math language is used.

No math reasoning is presented.

Apprentice

Big Scoop

If either of the Girls icecream is where to melt they would either over flow or not fill the cup Devon would over flow because the volume of her cone is 133.97333cm^3 as for mally's her wouldn't fill her cup her volume is 401.92cm^3 the containers are either to small or to big.

Some math language is used, but some work is not shown.

Part of the problem is addressed. The student does not find the volume of the ice cream so no comparisons can be made.

Practitioner

Each scoop = $\frac{268}{3} \text{ cm}^3$ $\frac{4}{3} \pi r^3$ $4 \times 3.14 \times 4 \times 4 \times 4 \div 3$
 The cone will hold 134 cm^3 $\pi r^2 h \div 3$ $3.14 \times 4 \times 4 \times 8 \div 3$
 The cylinder will hold 402 cm^3 $\pi r^2 H$ $3.14 \times 4 \times 4 \times 8$

~~If the ice cream melts in either container the container will~~ ~~IF~~

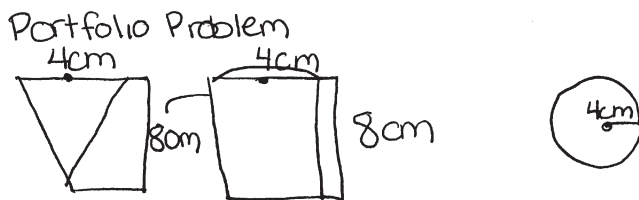
If the ice cream melts in the cone it will spill.
 If the ice cream melts in the cylinder it will be fine.

Correct answers are achieved.

All work is shown.

Accurate math language is used.

Expert



Volume of ice cream scoop = $\frac{4}{3}\pi 4^3 = 267.95 \text{ cm}^3$

Volume of the cone = $\pi 4^2 \cdot 8 \div 3 = 133.99 \text{ cm}^3$

Volume of the cylinder = $\pi 4^2 \cdot 8 = 401.92 \text{ cm}^3$

cylinder 401.92 cm^3

sphere -267.95 cm^3

$= 133.97 \text{ cm}^3$ extra space in the cylinder

. If the ice cream melted in the cylinder there would be 133.97 cm^3 extra space. If the ice cream melted in the cone the ice cream would overflow



Melted Ice cream

If you look at the top you will see my work. I first found out I need to find the volume of the cone, cylinder, and sphere. I used the formula $\frac{4}{3} \times 3.14 (\text{pie}) \times r^3$ to find the volume of the ice cream scoop. $3.14 (\text{pie}) \cdot r^2 (\text{radius}) \cdot h \div 3$ for the volume of the cone, and I used $3.14 (\text{pie}) \times r^2 \times h$ for a formula of a cylinder. The reason I used the formulas above is because they made sense for me to use.

Math language and representations are used to communicate the solution.

All work is shown and labeled.

Correct answers are achieved.

A mathematically relevant observation is made about the volume of the dish that is filled.