Basketball Packaging

The Bouncing Basketball Company is looking to design a new carton to ship 24 basketballs. Each basketball comes in a box that measures 1 foot on every side and they want to put 24 of these boxes in 1 carton. How many different ways can they design the carton so that it can contain the 24 basketball boxes? Which way do you think is the best? Mathematically explain your reasoning.

Basketball Packaging

Suggested Grade Span
Grades 6–8

Grade(s) in Which the Task Was Piloted
Grades 7 and 8

Task
The Bouncing Basketball Company is looking to design a new carton to ship 24 basketballs. Each basketball comes in a box that measures 1 foot on every side and they want to put 24 of these boxes in 1 carton. How many different ways can they design the carton so that it can contain the 24 basketball boxes? Which way do you think is the best? Mathematically explain your reasoning.

Alternative Versions of the Task

More Accessible Version:
The Bouncing Basketball Company is looking to design a new carton to ship 8 basketballs. Each basketball comes in a box that measures 1 foot on every side and they want to put 8 of these boxes in 1 carton. How many different ways can they design the carton so that it can contain the 8 basketball boxes? Which way do you think is the best? Mathematically explain your reasoning.

More Challenging Version:
The Bouncing Basketball Company is looking to design a new carton to ship 24 basketballs. Each basketball comes in a box that measures 1 foot on every side and they want to put 24 of these boxes in 1 carton. How many different ways can they design the carton so that it can contain the 24 basketball boxes? Which way do you think is the best? Mathematically explain your reasoning.

The basketballs themselves are 1 foot in diameter. How much of the volume of each carton will be made up of basketballs, and how much will be surrounding space?

NCTM Content Standards and Evidence

Geometry Standard for Grades 6–8

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Use visualization, spatial reasoning and geometric modeling to solve problems.
    - ***NCTM Evidence:*** Use two–dimensional representations of three–dimensional objects to visualize and solve problems such as those involving surface area and volume.
    - ***Exemplars Task Specific Evidence:*** This task requires students find the volume and surface area of basketball cartons.

Time/Context/Qualifiers/Tip(s) from Piloting Teacher

This is a medium length task. I used this as an assessment piece after completing the activity called "Packing the Packages" from *NCTM Student Notes*. Students should know the formula for surface area and volume to be most successful with solving this task.

Links

This task could link to technical education or art units involving designing containers. It could also accompany a unit in Physical Education on basketball.

Common Strategies Used to Solve This Task

Most students will generate the combinations using knowledge of factors. The student may use the surface area formula to determine the carton with the least amount of surface area. Students may also solve the problem through the understanding that as the shape of the carton is closer to a line, the surface area is greater, while as a shape approaches a cube, the surface area is smaller.

Possible Solutions

Original Version:

Dimensions of Carton With a Volume of 24′	Surface Area in Square Feet
24′ x 1′ x 1′	98
12′ x 2′ x 1′	76
8′ x 3′ x ‘1	70
6′ x 4′ x 1′	68
6′ x 2′ x 2′	56
4′ x 3′ x 2′	52

It is assumed that cartons be rectangular prisms, although some students may choose to be more creative.

The best shape box for the company to use would probably be the 3′ x 4′ x 2′. This shape is closest to a cube and will have the smallest amount of surface area. This means the company would have to pay the least amount for materials to make a carton with dimensions of 3′ x 4′ x 2′.

More Accessible Version:

Dimensions of Carton With a Volume of 24′	Surface Area in Square Feet
8′ x 1′ x 1′	34
4′ x 2′ x 1′	28
2′ x 2′ x 2′	24

More Challenging Version:

The original version, and…

Volume of a Sphere = $4\pi r^3 / 3$

$[4\ \pi\ (.5)^3]/3$ = .523598776 x 24 basketballs = 12.56637061 cubic feet

24 – 12.56637061 = 11.43362939 cubic feet extra

Task Specific Assessment Notes

General Notes: Most students will organize their data in a table and locate the shape that is closest to a cube. This task should elicit precise math language and representations.

Novice: The Novice will have no understanding of surface area and volume and will not have an approach to solve the problem.

Apprentice: The Apprentice will show some understanding of volume and surface area, but will not have all of the combinations, will have incorrect calculations of surface area, and/or will not answer all of the questions.

Practitioner: The Practitioner will show understanding of the concept of surface area and volume and will recognize that the shape of a cube will have the least amount of surface area. All computations will be correct and complete.

Expert: The Expert will show understanding of surface area and volume as it relates to this problem. The Expert will be able to generalize the known formula for surface area, or make other mathematically relevant comments or observations to extend the solution.

Author

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Novice

The student achieves no correct combinations.

Basketball Packaging

Our teacher gave us a portfolio that said… The Bouncing Basketball Company is looking to design a new carton to ship 24 basketballs. Each basketball comes in a box that measures one foot on every side and they want to put 24 of these boxes in one carton. How many different ways can they design the carton so that it can contain the 24 basketball boxes? Which way do you think it is the best? Mathematically explain your reasoning.

1,2,3,4,6,8,9,16,18,24,36,48,72,144

First I got all the numbers that are multiples of 144 because 24*6=144ft.which is the volume. After I did that I remembered a rule that we used in another problem and it was bh+hs+bs, which helped me, find the surface area or volume. The chart below shows the base, height, side, and volume of the cubes.

Base (B)	Height (H)	Length (L) or (S) Side	Volume (V)
1	1	144	144
2	72	1	144
3	48	1	144
4	36	1	144
6	24	1	144
8	18	1	144
9	16	1	144

I made a rule using bh+hs+bs=volume.

1*1*144=144
2*72*1=144
3*48*1=144
4*36*1=144
6*24*1=144
8*18*1=144
9*26*1=144

By using my formula my equations worked for the problem. The carton that has the dimensions of 2*72*1 is the best because it is the only one that could probably fit into a truck.

On the surface the student appears to have some understanding of the task, but it is superficial.

Apprentice

Work is organized and labeled.

Basketball Packaging

I was given a portfolio that asked me to try to find the most combinations of a volume of 24. I went about solving this problem by thinking what equals 24 so I did some guess and check. I came up with 5 combinations. To check my answer I multiplied the base, width, and height to see if it = 24.

Five out of six volume combinations are found. Surface areas are incorrect.

Base ft	Side ft	Height ft	Volume	S.A.
24	1	1	24	97
12	2	1	24	76
3	8	2	24	60
4	6	1	24	68
6	2	2	24	56

Here is how I found my answer for S.A by finding the rule of the class.

(2bs)+(2bh)+(2hs)=S.A.

Some understanding of the task is demonstrated.

Practitioner

Formulas are identified that assist in solving the task.

Correct solutions are achieved with supporting work.

Basketball Packaging

My class was given a problem that we needed to make a box that could hold 24 basketballs that had a one foot length on each side.

Now after I had gotten the portfolio I made a table, the table had length, width and height.

Length (L) feet	Width (W) feet	Height (H) feet	Surface Area (SA)feet²
8	3	1	70
24	1	1	98
6	4	1	68
12	2	1	76
6	2	2	56
4	3	2	52

Now this is how I got all my answers. I got 24 cubs and made all the combinations with them I knew they were right because I multiplied each combination together and they equaled 24. I found 6 different ways to organize the cubes. I used the surface area formula to fill the last column. The formula is (2WL)+(2HL)+(2WH).

(2x3x8)+(2x1x8)+(2x3x1)=70 feet²
(2x4x6)+(2x1x6)+(2x4x1)=68feet²
(2x2x12)+(2x1x12)+(2x2x1)=76feet²
(2x2x6)+(2x2x6)+(2x2x2)=56feet²
(2x3x4)+(2x2x4)+(2x3x2)=52feet²
(2x1x24)+(2x1x24)+(2x1x1)=98feet²

I think that they should use the box with the dimensions of length 4 width 3 height 2.

This is because it has the least surface area and it wouldn't take up a lot of room.

You should use this one also because it would be one of the easiest to carry.

You might want to get this because it is the cheapest out of all of them.

Math language and representations are used to clearly communicate the solution to the audience.

Expert

Accurate labels of measurement units are used.
Math language is precise.

Basketball Packaging

The math representation is well organized and labeled precisely.

We got a portfolio in class that asked us how many different prisms could we make to hold 24 basketballs in 1 x 1 x 1 boxes.

We had done a piece like this before so I knew how to go about it. I started by making a table to organize my data. Then I started coming up with combinations for the dimensions of the packages. The volume of the box was 24 basketballs, so to make sure my dimensions were correct I would have to multiply base by height by side. If it equaled 24 then I knew that could be one of my dimensions. Here is my table:

Dimensions Table

Base (b)ft	Height (h)ft	Side(s)ft	Volume ft^3	Surface Area ft^2
24	1	1	24	98 ft^2
12	2	1	24	~~80~~ 76 ft^2
8	3	1	24	70 ft^2
6	4	1	24	68 ft^2
6	2	2	24	5~~8~~6 ft^2
4	2	3	24	52 ft^2

To get the surface area I used a known rule. Which is the rule for surface area it is shown below. The rule works because rectangular prisms have 3 different sides that each have another side identical to it.

(2 x b x h) + (2 x h x s) + (2 x b x s) = Surface Area

Formulas are identified and used to solve the task. The student explains the derivation of the surface area formula.

A correct answer is achieved with supporting work.

Expert cont.

Here are some examples that my rule works. These cases come from my table above.

$$(2 \times 24 \times 1) + (2 \times 1 \times 1) + (2 \times 1 \times 24) = 98 \text{ ft}^2$$
$$(2 \times 12 \times 1) + (2 \times 1 \times 2) + (2 \times 12 \times 2) = 76 \text{ ft}^2$$

The portfolio also asked us which package was the best. That could mean a couple things if you wanted the box that would take up the least surface area. Then you would want the package closest to a cube, which is 4 x 2 x 3. If you wanted the package with the most surface area then you would want the one closest to a line, which would be 24 x 1 x 1.