

Miles Per Gallon

Mrs. Ima Driver's car gets 24 miles per gallon of gasoline and Mr. Yul B. Driver's truck gets 18 miles per gallon. When traveling from Burlington to White River they both use a whole number of gallons of gasoline. What is the shortest distance possible between Burlington and White River?

Miles Per Gallon

Suggested Grade Span

Grades 6–8

Grade(s) in Which the Task Was Piloted

Grades 7 and 8

Task

Mrs. Ima Driver's car gets 24 miles per gallon of gasoline and Mr. Yul B. Driver's truck gets 18 miles per gallon. When traveling from Burlington to White River they both use a whole number of gallons of gasoline. What is the shortest distance possible between Burlington and White River?

Alternative Versions of the Task

More Accessible Version:

Mrs. Ima Driver's car gets 24 miles per gallon of gasoline and Mr. Yul B. Driver's truck gets 18 miles per gallon. When traveling from Burlington to White River they both use a whole number of gallons of gasoline. What is one possible distance between Burlington and White River?

More Challenging Version:

Mrs. Ima Driver's car gets 24 miles per gallon of gasoline and Mr. Yul B. Driver's truck gets 18 miles per gallon. When traveling from Burlington to White River they both use a whole number of gallons of gasoline. What is the shortest distance possible between Burlington and White River? If gasoline costs \$1.49 per gallon, what is the difference in price each will spend on a round trip?

NCTM Content Standards and Evidence

Number and Operations Standard for Grades 6–8

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
 - *NCTM Evidence:* Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.
 - *Exemplars Task Specific Evidence:* This task requires students find the least common multiple of the miles per gallon of each vehicle.

Time/Context/Qualifiers/Tip(s) from Piloting Teacher

This is a short task as it took less than one 45-minute class period to complete.

The class had finished a fraction unit and was studying time–rate–distance problems. The task was given to assess if students recognized a Least Common Multiple situation.

Links

This task could link to a consumer unit where students might compare the different mileage rates of different automobiles to see which would be most economical. Students could also investigate the cost of owning a car for a 1-year period of time.

Common Strategies Used to Solve This Task

Most students found multiples of one of the numbers and checked to see if the other divided into the product.

Possible Solutions

Original Version:

Seventy-two miles is the smallest possible distance between Burlington and White River.

More Accessible Version:

Any common multiple of 24 and 18 is acceptable.

More Challenging Version:

The original version and...

Mrs. Ima: $72 \div 24 = 3$ gallons \times $\$1.49 = \4.47×2 (round trip) = $\$8.94$

Mr. Yul: $72 \div 18 = 4$ gallons \times $\$1.49 = \5.96×2 (round trip) = $\$11.92$

Difference $\$2.98$

Task Specific Assessment Notes

General Notes: The task lends itself to assessing understanding of Least Common Multiple.

Novice: The Novice will not understand the need to use only the information in the problem. The Novice will try to introduce other elements into the task such as size of gas tanks or maps. The Novice may attempt to find multiples with no understanding of how to proceed.

Apprentice: The Apprentice may recognize the need to find a common number of miles and the need to find the smallest common multiple, but does not have an accurate procedure for doing so.

Practitioner: The Practitioner will recognize the need to find the Least Common Multiple and finds the correct solution. Accurate math language and representations will be used to communicate the solution.

Expert: The Expert recognizes this as a Least Common Multiple situation and has an efficient strategy to solve the task. The Expert will find the correct solution. The Expert may also discuss more than one strategy to find LCM, or may state a rule for solving the problem.

Author

Al Bianchi, 6–12 Mathematics Assessment Consultant at the Vermont Department of Education, wrote this task.

Exemplars

The student's limited understanding of the task does not lead toward a correct solution.

Novice

Miles per gallon
 TO FIND HOW FAR IT IS
 FROM BURLINGTON TO
 White River. I know that
 the car gets 24 miles
 per gallon and the
 TRUCK gets 18 miles
 per gallon. The car
 tank holds 15 gallons
 of gas and the TRUCK
 tank holds 20 gallon
 of gas

SO multiply miles per
 gallon BY gallon in
 tank

$$\begin{array}{r} 24 \\ \times 15 \\ \hline 120 \\ 360 \\ \hline \end{array} \quad \begin{array}{r} 18 \\ \times 20 \\ \hline 360 \\ \hline \end{array}$$

$\frac{24}{360}$ miles per gallon 6 gallon distance
 $\frac{18}{360}$ miles per gallon 20 gallon distance

CAR	24	15	360
TRUCK	18	20	360

It is unclear how the student determined the number of gallons in the tank.

Exemplars

Apprentice

The student demonstrates some understanding of the underlying mathematics necessary for solving the task.

The student is able to find common multiples, but not the least common multiple.

Miles Per Gallon

I need to find the distance from Burlington to White River if I know the car gets 24 miles per gallon and a truck gets 18 miles per gallon. Both use a whole number of gallons.

I need to find a number that can be divided by both 24 and 18 evenly.

$$\begin{array}{r} 24 \\ \times 18 \\ \hline 192 \\ 24 \\ \hline 432 \end{array}$$

$$\begin{array}{r} 432 \div 18 \\ \underline{24} \\ 18 \overline{)432} \\ \underline{36} \\ 72 \\ \underline{72} \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ 24 \overline{)432} \\ \underline{24} \\ 192 \\ \underline{192} \\ \hline \end{array}$$

Since 432 can be divided by both 24 and 18, that is the distance between Burlington and White River but this seems too much. So I tried some other number and I found 144 works.

$$\begin{array}{r} 8 \\ 18 \overline{)144} \\ \underline{144} \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ 24 \overline{)144} \\ \underline{144} \\ \hline \end{array}$$

So I think 144 miles is a better answer and 144 is smaller than 432

Some work is shown.
Communication is clear.

Practitioner

A correct answer is achieved although not efficiently. Work is shown, labeled and organized.

Miles Per Gallon

I have to find the distance from Burlington to White River. I have to find this using math and cannot look on a map.

What I Know.

Vehicle	Miles Per Gallon (mpg)
Car	24
Truck	18

I also know that both the car and the truck use a whole number of gallons of gas.

How I solved this.

Number of Gallons	Car Distance in miles	Truck Distance in miles
1	24 x 1 = 24	18 x 1 = 18
2	24 x 2 = 48	18 x 2 = 36
3	24 x 3 = 72	18 x 3 = 54
4	24 x 4 = 96	18 x 4 = 72
5	24 x 5 = 120	18 x 5 = 90
6	24 x 6 = 144	18 x 6 = 108
7	24 x 7 = 168	18 x 7 = 126
8	24 x 8 = 192	18 x 8 = 144

Relevant observations are made about the underlying mathematics in the task.

What I Noticed.

72 and 144 are in both columns of my chart. So the smallest distance is 72 miles.

This problem is like trying to find a number that can be divided by two other numbers.

Example What number can be divided by both 6 and 8 evenly. 6 x 4 = 24 and 8 x 3 = 24 so 24 is the smallest number that can be divided by both 6 and 8 evenly.

Exemplars

Expert

Precise math language is used to communicate. Representations assist in communicating the approach, reasoning and solution.

The student identifies the underlying mathematics in the task, allowing for "generalizability" of the solution.

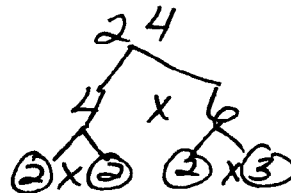
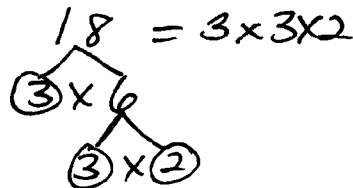
Miles Per Gallon

I have to find out mathematically how many miles it is from Burlington to White River. I cannot use a map. I know the miles per gallon rate for a car and a truck.

Gas Usage	
Car	24 miles per gallon
Truck	18 miles per gallon

I also am given the information that both cars use a whole number of gallons.

I think that this is like those problems that you are trying to find a number that can be divided evenly by 2 other numbers, or like finding a common denominator in fractions. That procedure was called finding the Least Common Multiple (LCM). The easiest way that I learned to do this is by using a ^{Diagram} tree. Just I need to find the prime factors of each number.



numbers in circles are prime numbers.

next I will make that diagram which shows what factors are shared and which are not.

The student communicates directly and clearly with the audience.

