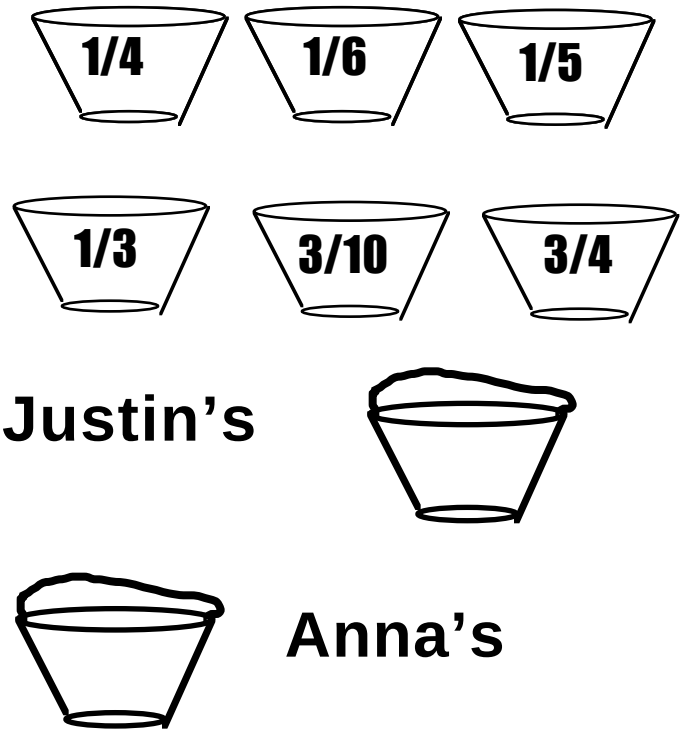
Lugging Water I

Justin and Anna were camping with their family. They joined their dad at the camp water–pump where he had partially filled 6 containers. The containers had no handles. As he filled each one, he labeled the fractional amount to which each container was filled. The amounts are shown below.



Justin and Anna each had a container that was the same size as the ones their dad filled, but theirs had handles. Their task was to pour the water from the 6 containers into their 2 containers so they could easily carry the water back to camp. Which containers should Justin and Anna pour into each of their containers so together they can transport the water in one trip? Show your math thinking.

Lugging Water I

Suggested Grade Span
Grades 3–5

Grade(s) in Which Task Was Piloted
Grade 5

Task
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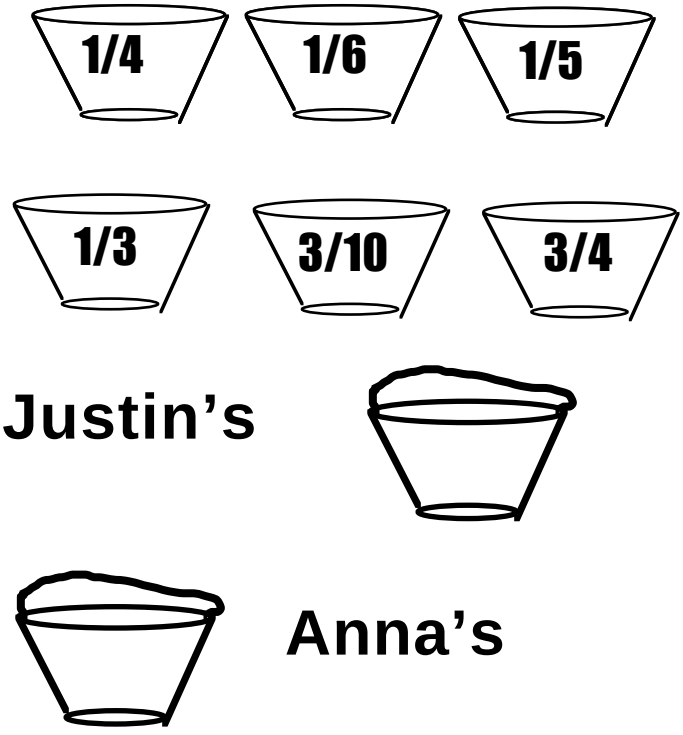

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Alternative Versions of Task

More Accessible Version:
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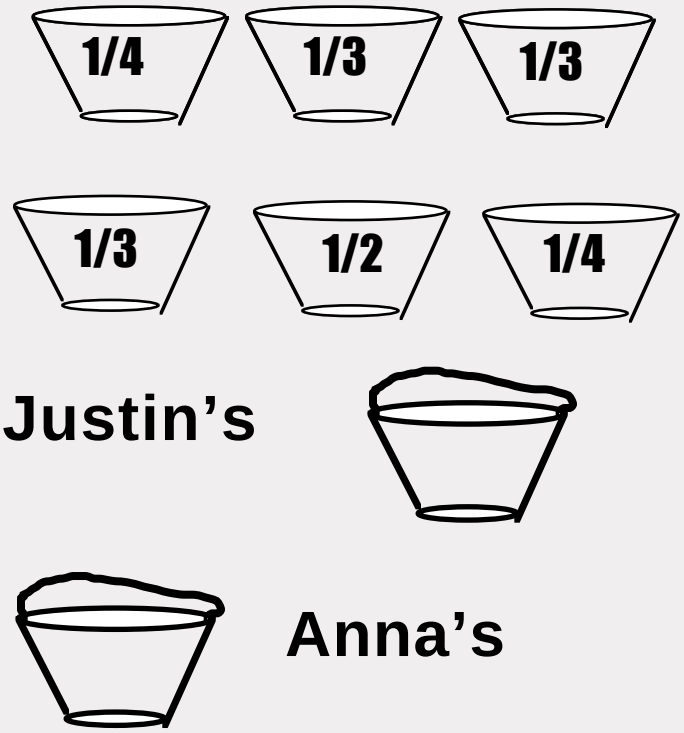

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More Challenging Version:
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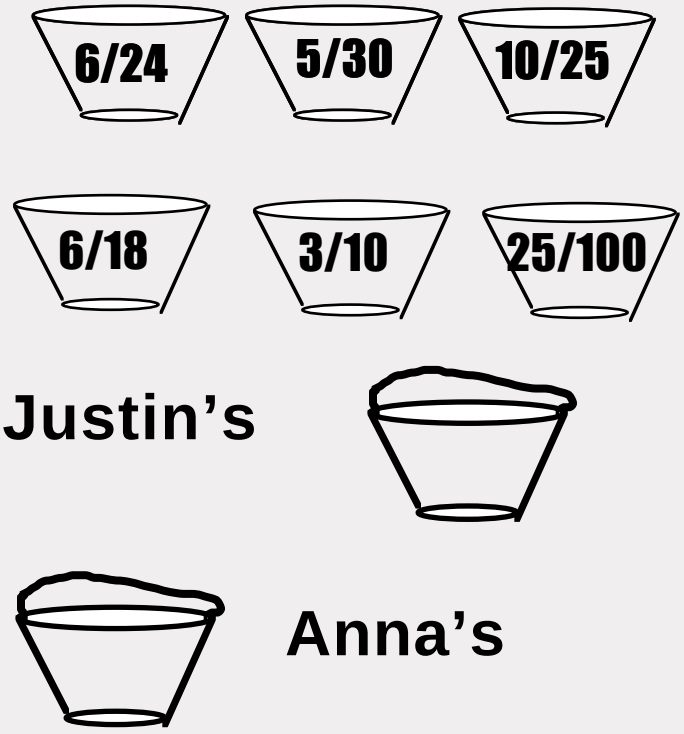

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NCTM Content Standards and Evidence

Number and Operation Standard for Grades 3–5

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Compute fluently and make reasonable estimates.
  - ***NCTM Evidence:*** Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;

- Use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals.
    - ***Exemplars Task Specific Evidence:*** This task requires students to find combinations of fractions with unlike denominators that have a sum equal to no more than a whole.

Time/Context/Qualifiers/Tip(s) From Piloting Teacher
This is a medium length task for most students. Students who have a command of addition of fractions with unlike denominators may not find this as a situation where problem solving strategies need to be employed, and should be given a different task to solve.

Links
This task could link to a unit on water, or to camping.

Common Strategies Used to Solve This Task

Most students will try to rename fractions so that they all have like denominators. Others will rename the fractions as decimals or percents. Then students guess and check to find combinations that are less than or equal to 1.

Possible Solutions

Original Version:
Justin's container = 1/4 + 3/4 = 1 whole
Anna's container = 1/6 + 1/5 + 1/3 + 3/10 = 1 whole

More Accessible Version:
Justin's container = 1/4 + 1/4 + 1/2 = 1 whole
Anna's container = 1/3 + 1/3 + 1/3 = 1 whole

More Challenging Version:

6/24 = 25%	5/30 = 16.666%	10/25 = 40 %
6/18 = 33.333%	3/10 = 30%	25/100 = 25 %

There are many different possible combinations. 25% + 16.666% + 40% + 33.333% + 30% + 25% = 169.999% so any one bucket may not contain more than 84.999% (169.999% ÷ 2) or the other will overflow.

Task Specific Assessment Notes

General Notes: This task should elicit the use of math language to solve and communicate a solution. It will also be important for students to communicate clearly with the audience. Students command of fractions in a problem solving setting can also be evaluated.

Novice: The novice will demonstrate little or no understanding of the task. The novice will not know that the fractions need to be changed into equivalent forms so as to make computation more manageable. Little engagement in the task will be evident, or all work present will be incorrect.

Apprentice: The apprentice will have a partially correct solution with a strategy that will work for solving part of the task. The apprentice may achieve a correct solution without mathematical justification. This will especially be the case where students choose to use diagrams to solve the task. If diagrams are used they need to be labeled and done with intent to show that the sum of the fractions does not exceed a whole. The apprentice may have an approach that would work for solving the task, but achieves an incorrect solution due to computation or reasoning errors.

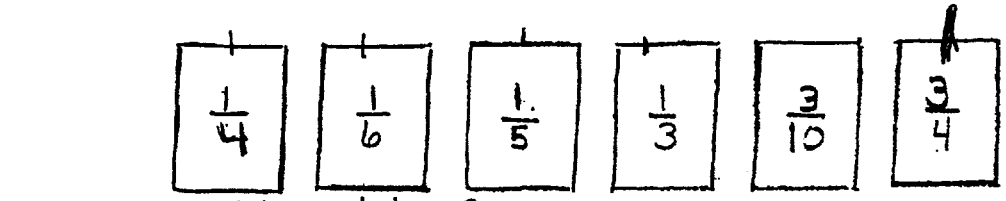
Practitioner: The practitioner will achieve a correct solution to the task with supporting work. The practitioner will clearly communicate the solution with correct reasoning and justification. Math language will be relied on to communicate to the audience.

Expert: The expert will not only achieve a correct solution, but will also utilize other good problem solving strategies such as creating a rule to solve the task, or going above and beyond the task requirements to extend the solution. Many students may rely on verifying the solution in order to prove their solution is correctness.

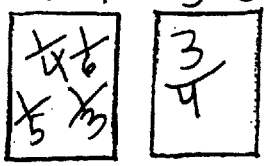
Author

This task was written by **Deb Armitage**, Pre K–8 Mathematics Assessment Consultant at the Vermont Department of Education. The task was piloted by teachers and students in Vermont.

Novice



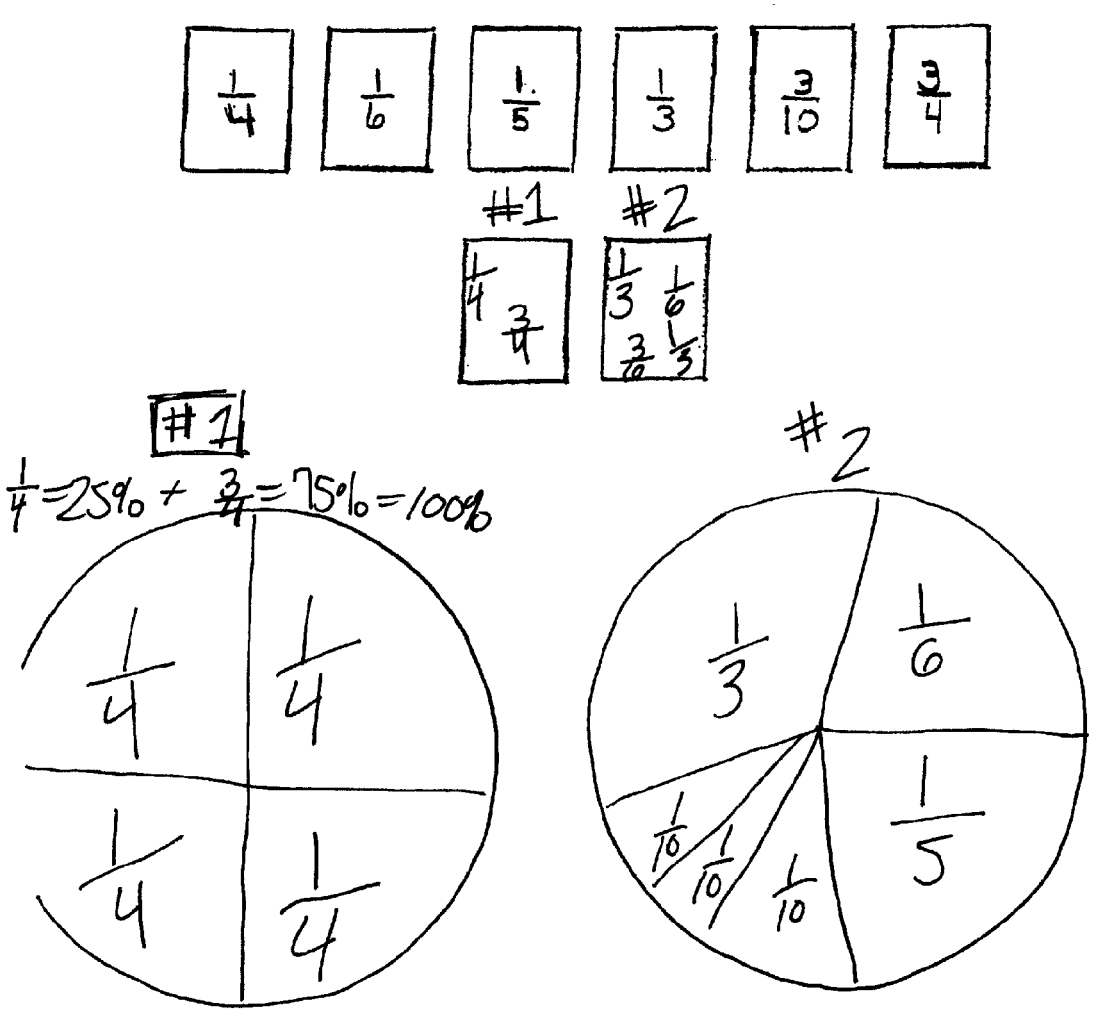
I take to find which fraction gose in wich contain



I will use guessand Check

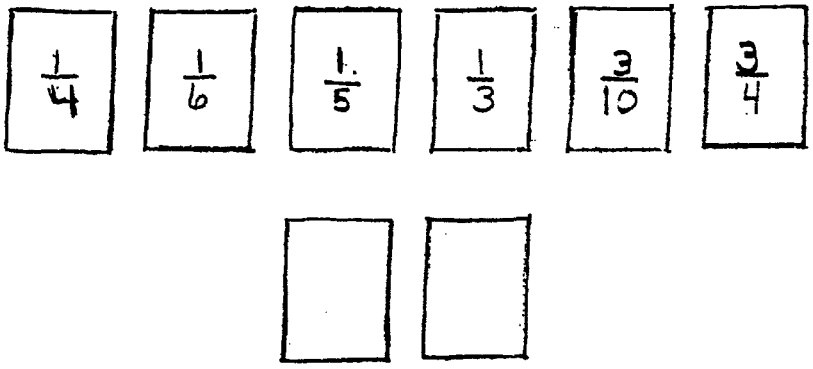
The student used "guess" as a method of finding an answer. But did not use a problem solving strategy to "click." The student also neglects to include 3/10 in the answer shown.

Apprentice



The student shows how container #1 can contain the 1/4 bucket and the 3/4 bucket, but the student's diagram for container #2 lacks preciseness and it is not accompanied by supporting work.

Practitioner



I have to use the fractions of water hat can go into the 2 empty container I will try different combinations percentagesto try and guess the answers.

A correct answer is achieved. Work is shown, labeled, and explained. Math language of fractions and percents is relied on for communication.

Practitioner cont.

to get % divide numerator by denominator

1 Container

$\frac{1}{4} = 25\%$

$\frac{3}{4} = 75\%$

$\frac{4}{4}$ 100%

2 Container

$\frac{1}{6} = 16\frac{2}{3}\%$

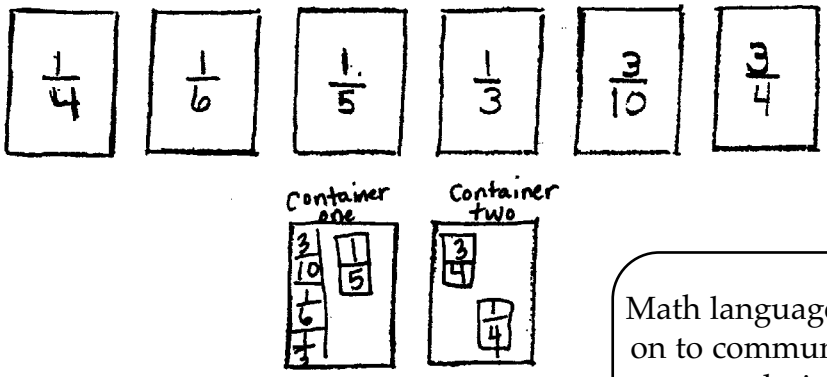
$\frac{1}{5} = 20\%$

$\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{3}{10} = 30\%$

100%

Expert



Math language is relied on to communicate the solution.

Guess 1: $\frac{3}{10}+\frac{1}{6}+\frac{1}{5}+\frac{1}{3}=$?

$\frac{9}{30}+\frac{5}{30}+\frac{6}{30}+\frac{10}{30}=\frac{30}{30}$ 1 whole now I will prove this with frac with fraction circles on the back.

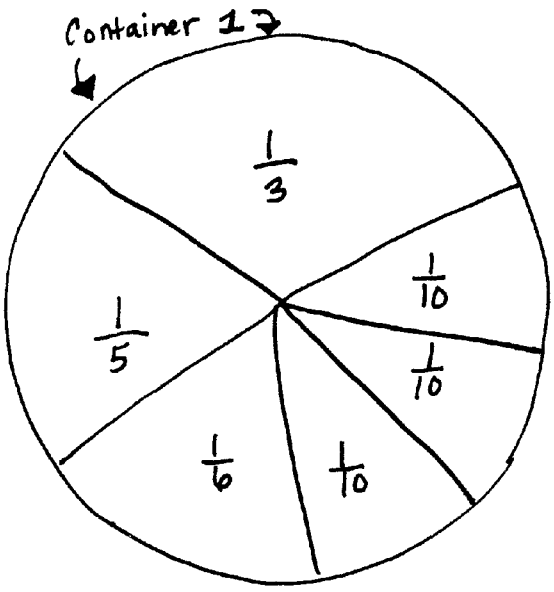
Guess 2: $\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$ 1 whole now I will prove this fraction with fraction circles on the back.

* diagrams on back →

A correct answer is achieved. Work is clearly shown and labeled.

The student shows a command of fractions concepts and skills.

Expert cont.



The student verifies the computation using diagrams.