Pyramids of Giza

The Pyramids of Giza stand on the west bank of the Nile River outside Cairo, Egypt. There are 10 of them, including 3 of the largest, best preserved pyramids in Egypt.

The pyramid of Khufu, called the *Great Pyramid*, contains more than 2 million stone blocks that average 2.3 metric tons each. The pyramid was originally 481 feet tall, but some of its upper stones are gone, and now it stands about 450 feet high. Its base covers about 13 acres (Note: 1 acre = 43,560 square feet).

Scholars believe that it took about 100,000 men to build the pyramid, and it took about 4 months to complete.

If this is the case, on average, how many cubic feet of pyramid were built by each worker per day?

The Great Pyramid, built about 4,500 years ago, rises at Giza, near Cairo.

Pyramids of Giza

Suggested Grade Span
Grades 6–8

Grade in Which Task Was Piloted
Grade 8

Task
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Alternative Versions of Task

More Accessible Version:
The pyramid of Khufu, called the *Great Pyramid*, contains more than 2 million stone blocks that average 2.3 metric tons each. The pyramid was originally 481 feet tall. Its base covers about 566,280 square feet.

Scholars believe that it took about 120 days to complete.

If this is the case, on average, how many cubic feet of pyramid were built each day?

More Challenging Version:
The Pyramids of Giza stand on the west bank of the Nile River outside Cairo, Egypt. There are 10 of them, including 3 of the largest, best preserved pyramids in Egypt.

The pyramid of Khufu, called the *Great Pyramid*, contains more than 2 million stone blocks that average 2.3 metric tons each. The pyramid was originally 481 feet tall, but some of its upper stones are gone, and now it stands about 450 feet high. Its base covers about 13 acres (Note: 1 acre = 43,560 square feet).

Scholars believe that it took about 100,000 men to build the pyramid, and it took about 4 months to complete.

If this is the case, on average, how many cubic feet of pyramid were built by each worker per day?

Choose another famous pyramid and compare its size to that of the Great Pyramid. Compare the number of days it took to build both pyramids as well as the number of workers.

NCTM Content Standards and Evidence

Geometry Standard for Grades 6–8

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - ***NCTM Evidence:*** Use two–dimensional representations of three–dimensional objects to visualize and solve problems such as those involving surface area and volume.

  - ***Exemplars Task Specific Evidence:*** This task requires students to visualize a 3 –dimensional pyramid from a 2–dimensional picture.

Measurement Standard for Grades 6–8

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Apply appropriate techniques, tools, and formulas to determine measurements.
  - ***NCTM Evidence:*** Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders.
  - ***Exemplars Task Specific Evidence:*** This task requires students to find the volume of a pyramid.

Number and Operation Standard for Grades 6–8

Instructional programs from Pre–Kindergarten through grade 12 should enable students to...

- Compute fluently and make reasonable estimates.
  - ***NCTM Evidence***: Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.
  - ***Exemplars Task Specific Evidence:*** This task requires students to multiply and divide to find the amount of work done by each person per day.

Time/Context/Qualifiers/Tips From Piloting Teacher

This is a medium length task. It took one to two class periods to complete.

Links

This task would link well to studies of ancient times, pyramids, Egypt, Mexico, or other countries where pyramids appear.

Common Strategies Used to Solve the Task

Most students begin by determining the area of the base of the pyramid, and then use this information to determine the volume of the pyramid. Next students divide the volume by the number of days worked. That is then divided by the number of men. This results in the work done per person per day.

Possible Solutions

Original Version:

Volume of Pyramid = [area of base x height] ÷ 3
43560 x 13 acres = 566,280 square feet.

566,280 square feet x 481 feet high = 272,380,680 ÷ 3 = 90,793,560 cubic feet.

4 months x average of 30 days in a month = 120 days in all.

90,793,560 cubic feet ÷ 120 days = 756,613 cubic feet a day.
756,613 cubic feet a day ÷ 100,000 men = 7.56613 cubic feet per person per day.

More Accessible Version:
566,280 square feet x 481 feet high = 272,380,680 ÷ 3 = 90,793,560 cubic feet.
90,793,560 cubic feet ÷ 120 days = 756,613 cubic feet a day.

More Challenging Version:
See above. Correctness of solution of the second part of the task will need to be assessed on an individual basis.

Task Specific Assessment Notes

General Notes: This task lends itself to using a variety of math language, but does not lend itself well to making math representations. Student should not be penalized for not using one.

Novice: The novice will make no significant progress toward a solution. Parts will be unclear, and many errors will be present.

Apprentice: The apprentice will make progress toward solving part of the task. The apprentice may have an approach that would work, but does not follow through correctly on the approach due to omission, reasoning or computation errors.

Practitioner: The practitioner will have a correct solution that is well documented, organized, and clear.

Expert: The expert will not only have a correct solution with supporting work, but will also go above and beyond task requirements creating a rule for solving the task, verifying the solution, or making mathematically relevant observations that are used to extend the solution.

Author

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Novice

43,600
× 13
566,280 feet²
× 450 feet
254,826,000

No significant part of the solution is correct. The student neglects to divide the base times height by 3.

254,826,000 total area
÷ 2,000,000
127 cubic feet
÷ 4 months
31.75 cubic feet

÷ 127
÷ 12
cubic feet a day

we found how much feet are in 66 13 acrrs xwe got a tonall of 566,280 then we times it by 480 seet equals 339,826.000 then we tolae that nzmber and divided it by 2 million and got 122 than i tne to average of 30 days in a manth than times it by 4 and got 124 Then divided 127 by 124 and got 1 cubic feet a

It is unclear why the student divides their "volume" by two million and why they divided by 127. Cumputation errors are present.

Apprentice

1. volume of pyrimid
$$\frac{\text{area of base} \times \text{height}}{3}$$

2. volume ÷ 100,000 =
work per person in all

3. Take answer to #2 and divide by the number of days.

Choose
ave month
= 30 days
× 4
120 days

This apprentice listed the correct steps to solving the task but did not attempt to execute them. Some math language is used to communicate.

Practitioner

base = 13 acres x 43,560 square feet
Base = 566280 square feet

height = 481 ft

volume of pyramid $\frac{b \times h}{3}$
volume = 90793560 ft3
volume ÷ 100,000 work per person in all
work per person for all = 907.93560 ft3
take answer from 2 ÷ by days
days = 30.26452
chose avg. month = 30 days
X 4
120 days

90,793,560 ÷ 120 = 756.613 ft3 built per day

756.663 ÷ 100,000 = 7.56613 ft3
work done

Connection

volume of blocks
V = 4539.679 ft3

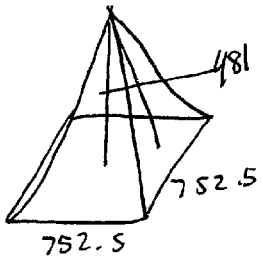
90793560 ft3 ÷ 2 million

A correct solution is achieved. All work is shown and labeled. Precise math language is used to communicate.

Expert

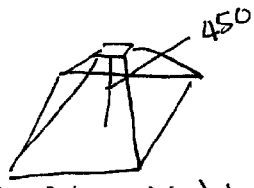
43,560 ft²
× 13 acres
566,280 ft²
× 481 height
÷ 3
V = 90,793,560 ft³

$V = \frac{\text{area of Base} \times \text{height}}{3}$



30 days × 4 months = 120 days

90793560
÷ 120 days
756,613 ft³
÷ 100,000
7.56613 ft³ – work done per person



Frustum $V = \frac{1}{3}h[\text{area of base 1} + \text{area base 2} + \sqrt{(a_1 a_2)}]$

connection

to find the volume now that the upper stones are missing you would have to use formula for a frustum. Since we don't know area of base 2 I can't figure it out.

A correct answer is achieved. All work is shown, labeled and organized. Precise math language is used throughout the solution to communicate.

The student extends the solution by applying prior math knowledge.